TOPIC = VECTOR, 3D AND MIX PROBLEMS (COLLECTION # 1)

Single Correct Type

Que. 1. The number $N = 6\log_{10} 2 + \log_{10} 31$, lies between two successive integers whose sum is equal to

(a) 5 (b) 7 (c) 9 (d) 10 **(code-V1T2PAQ1)**

Que. 2. Number of integers satisfying the inequality $\log_{1/2}|x-3| > -1$ is

(a) 5 (b) 3 (c) 2 (d) infinite **(code-V1T4PAQ1)**

Que. 3. $10^{\log_p(\log_q(\log_r x))} = 1$ and $\log_q\left(\log_r\left(\log_p x\right)\right) = 0$ then 'p' equals

(a) $r^{q/r}$ (b) rq (c) 1 (d) $r^{r/q}$ **(code-V1T4PAQ7)**

Que. 4. $a = \log 12,\ b = \log 21,\ c = \log 11,$ and $d = \log 22$ then $\log\left(\frac{1}{7}\right)$ can be expressed in this form $P(a-b)+Q(c-d)$ where P and Q are integers then the value of $(7P-Q)$ equals

(a) 5 (b) 9 (c) 13 (d) 15 **(code-V1T5PAQ3)**

Que. 5. If the equation, $x^{\log_a x^2} = \frac{x^{k-2}}{a^k}, a > 0, a \neq 0,$ has has exactly one solution for x, then the sum of the two possible values of 'k' is **(code-V1T5PAQ10)**

(a) 4 (b) 10 (c) 12 (d) $8\sqrt{2}$

Que. 6. There exist a positive number k such that $\log_2 x + \log_4 x + \log_8 x = \log_k x,$ for all positive real numbers x. If $k = \sqrt[b]{a}$ where $a, b \in N$, the smallest possible value of $(a+b)$ is equal to **(code-V1T7PAQ3)**

(a) 75 (b) 65 (c) 12 (d) 63

Que. 7. Suppose that a,b,c,d are real numbers satisfying $a \geq b \geq c \geq d \geq 0, a^2+d^2=1; b^2+c^2=1$ and $ac+bd=1/3$. The value of $(ab-cd)$ is equal to **(code-V1T7PAQ9)**

(a) $\frac{2}{3\sqrt{2}}$ (b) $\frac{2\sqrt{2}}{3}$ (c) $\pm\frac{2\sqrt{2}}{3}$ (d) $\frac{\sqrt{3}}{2}$

Que. 8. Let x and y be numbers in the open interval (0, 1). Suppose there exists a positive number'a', different from 1 such that $\log_x a + \log_y a = 4\log_{xy} a$. Which of the following statements are necessary true ? **(code-V1T10PAQ1)**

I $(\log_a x + \log_a y)^2 = 4\log_a x \log_a y$

II $x = y$

III $\log_{x^2} a = \log_x \sqrt{a}$

(a) I only (b) II only (c) I and II only (d) I, II and III

Que. 9. Let T, E, K and O be positive real numbers such that $\log(T.O)+\log(T.K)=2;\ \log(K.O)+\log(K.E)=3;$ $\log(E.T)+\log(E.O)=4$ The value of the product (TECO) equals (base of the log is 10)

(a) 10^2 (b) 10^3 (c) 10^4 (d) 10^9 **(code-V1T12PAQ4)**

Que. 10. If P is the number of natural numbers whose logratihms to the base 10 have the characteristic p and Q is the number of natural numbers logarithms of whose reciprocals to the base 10 have the characteristic –q then $\log_{10} P - \log_{10} Q$ has the value equal to **(code-V1T13PAQ24)**

(a) $p - q + 1$ (b) $p - q$ (c) $p + q - 1$ (d) $p - q - 1$.

Que. 11. P be a point interior to the acute triangle ABC. If $\overrightarrow{PA} + \overrightarrow{PB} + \overrightarrow{PC}$ is a null vector then w.r.t. the trangle ABC, the point P is, its **(code-V2T8PAQ3)**

(a) centroid (b) orthocentre (c) incentre (d) corcumcentre

Que. 12. L_1 and L_2 are two lines whose vector equations are **(code-V2T9PAQ1)**

$$L_1 : \vec{r} = \lambda\left(\left(\cos\theta + \sqrt{3}\right)\hat{i} + \left(\sqrt{2}\sin\theta\right)\hat{j} + \left(\cos\theta - \sqrt{3}\right)\hat{k}\right)$$
$$L_2 : \vec{r} = \mu\left(a\hat{i} + b\hat{j} + c\hat{k}\right),$$

where λ and μ are scalars and α is the acute angle between L_1 and L_2. If the angle 'α' is independent of θ then the value of 'α' is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

Que. 13. Which of the following are equation for the plane passing through the points $P(1,1,-1), Q(3,0,2)$ and $R(-2,1,0)$? **(code-V2T9PAQ2)**

(a) $\left(2\hat{i} - 3\hat{j} + 3\hat{k}\right).\left((x+2)\hat{i} + (y-1)\hat{j} + z\hat{k}\right) = 0$

(b) $x = 3 - t,\ y = -11t,\ z = 2 - 3t$

(c) $(x+2) + 11(y-1) = 3z$

(d) $\left(2\hat{i} - \hat{j} - 2\hat{k}\right) \times \left(-3\hat{i} + \hat{k}\right).\left((x+2)\hat{i} + (y-1)\hat{j} + z\hat{k}\right) = 0$

Que. 14. If $\vec{u}$ and $\vec{v}$ are two vectors such that $|\vec{u}| = 3; |\vec{v}| = 2$ and $|\vec{u} \times \vec{v}| = 6$ then the correct statement is **(code-V2T9PAQ3)**

(a) $\vec{u} \wedge \vec{v} \in (0, 90^\circ)$ (b) $\vec{u} \wedge \vec{v} \in (90^\circ, 180^\circ)$

(c) $\vec{u} \wedge \vec{v} = 90^\circ$ (d) $(\vec{u} \wedge \vec{v}) \times \vec{u} = 6\vec{v}$

Que. 15. $P(\vec{p})$ and $Q(\vec{q})$ are the positive vectors of two fixed points and $R(\vec{r})$ is the position vector of avariable point. If R moves such that $(\vec{r} - \vec{p}) \times (\vec{r} - \vec{q}) = 0$ then the locus of R is **(code-V2T12PAQ1)**

(a) a plane containing the origin 'O' and parallel to two the non collinear vectors $\overrightarrow{OP}$ and $\overrightarrow{OQ}$
(b) the surface of a sphere described on PQ as its diametere.
(c) a line passing through the points P and Q
(d) a set of lines parallel to the line PQ.

Que. 16. The range of values of m for which the line $y = mx$ and the curve $y = \frac{x}{x^2+1}$ enclose a region, is

(a) $(-1,1)$ (b) $(0,1)$ (c) $[0,1]$ (d) $(1, \infty)$ **(code-V2T17PAQ3)**

Comprehesion Type

1 Paragraph for Q. 1 to Q. 3

Vertices of a parallelogram taken in order are $A(2,-1,4); B(1,0,-1); C(1,2,3)$ and D.

1. The distance bertween the parallel lines AB and CD is **(code-V2T9PAQ4,5,6)**

(a) $\sqrt{6}$ (b) $3\sqrt{\frac{6}{6}}$ (c) $2\sqrt{2}$ (d) 3

2. Distance of the point $P(8,2,-12)$ form the plane of the parallelogram is

(a) $\frac{4\sqrt{6}}{9}$ (b) $\frac{32\sqrt{6}}{9}$ (c) $\frac{16\sqrt{6}}{9}$ (d) None

3. The areas of the orthogonal projections of the parallelogram on the three coordinates planes xy, yz and zx respectively

(a) 14, 4, 2 (b) 2, 4, 14 (c) 4, 2, 14 (d) 2, 14, 4

2 Paragraph for Q. 4 to Q. 6

The sides of a triangle ABC satsfy the relations $a+b-c=2$ and $2ab-c^2=4$ and $f(x)=ax^2+bx+c$.

4. Area of the triangle ABC in square units, is **(code-V2T15PAQ1,2,3)**

(a) $\sqrt{3}$ (b) $\frac{\sqrt{3}}{4}$ (c) $\frac{9\sqrt{3}}{4}$ (d) $4\sqrt{3}$

5. If $x \in [0,1]$ then maximum value of f(x) is

(a) 3/2 (b) 2 (c) 3 (d) 6

6. The radius of the circle opposite to the angle A is

(a) 1 (b) $\sqrt{3}$ (c) $\sqrt{3}/2$ (d) $1/\sqrt{3}$

Assertion & Reason Type

In this section each que. contains STATEMENT-1 (Assertion) & STATEMENT-2(Reason).Each question has 4 choices (A), (B), (C) and (D), out of which **only one is correct.**

Bubble (A) STATEMENT-1 is true, STATEMENT-2 is True; STATEMENT-2 is a correct explanation for STATEMENT-1.

Bubble (B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is NOT a correct explanation for STATEMENT-1.

Bubble (C) STATEMENT-1 is True, STATEMENT-2 is False.

Bubble (D) STATEMENT-1 is False, STATEMENT-2 is True.

Que. 1. Consider the following statements **(code-V1T2PAQ7)**

Statement - 1:

Number of cyphers after decimal before a significant figure comes in $N = 2^{-100}$ is 30.

because

Statement - 2:

Number of cyphers after decimal before a significant figure comes in $N = 2^{-10}$ is 3.

Que. 2 Consider the following statements **(code-V1T2PAQ8)**

Statement - 1 :

There exists some value of θ forwhich $\sec\theta = \frac{1}{2}\left(\log_{\frac{1}{\pi}} 7 + \log_7\left(\frac{1}{\pi}\right)\right)$

because

Statement - 2

If y is negative then $\frac{1}{2}\left(y+\frac{1}{y}\right) \le -1$

Que. 3. Statement - 1 : **(code-V1T4PAQ10)**

$\sqrt{\log_x \cos(2\pi x)}$ is a meaningful quantity only if $x \in (0,1/4)\cup(3/4,1)$.

because

Statement - 2 :

If the number N > 0 and the base of thelogarithm b (greater then zero not equal to 1) both lie on the same side of unity then $\log_b N < 0$ and it they lie on differnt side of unity then $\log_b N < 0$

Que. 4. Statement - 1 : If $N = \left(\frac{1}{0.4}\right)^{20}$ then N contains 7 digitis before decimal. **(code-V1T10PAQ8)**

because

Statement - 2 : Characteristic of the logarithm of N to the base 10 is 7. [use $\log_{10} 2 = 0.3010$]

Que. 5. Consider the following statements **(code-V1T12PAQ10)**

Statement - 1 : The equation $5^{\log_5(x^3+1)} - x^2 = 1$ has two distinct real solutions.

because

Statement - 2 : $a^{\log_a N} = N$ when $a > 0, a \ne 1$ and $N > 0$

Que. 6. Statement - 1 : $\log_{10} x < \log_\pi x < \log_e x < \log_2 x \,(x > 0 \text{ and } x \ne 1)$ **(code-V1T14PAQ4)**

because

Statement - 2 : If $0 < x < 1$, then $\log_x a < \log_x b \Rightarrow 0 < a < b$

Que. 7. Given lines $\frac{x-4}{2} = \frac{y+5}{4} = \frac{z-1}{-3}$ and $\frac{x-2}{1} = \frac{y+1}{3} = \frac{z}{2}$ **(code-V2T9PAQ7)**

Statement 1: The lines intersect.

because

Statement 2: They are not parallel.

Que. 8. Consider three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ **(code-V2T9PAQ8)**

Statement 1: $\vec{a}\times\vec{b} = \left(\left(\hat{i}\times\vec{a}\right).\vec{b}\right)\left(\left(\hat{j}\times\vec{a}\right).\vec{b}\right)\hat{j} + \left(\left(\hat{k}\times\vec{a}\right).\vec{b}\right)\hat{k}$

because

Statement 2: $\vec{c} = \left(\hat{i}.\vec{c}\right)\hat{i} + \left(\hat{j}.\vec{c}\right)\hat{j} + \left(\hat{k}.\vec{c}\right)\hat{k}$

More than One May Correct Type

Que. 1. In which of the following case(s) the real number 'm' greater than the real number 'n' ?

(a) $m = (\log_2 5)^2$ and $n = \log_2 20$ **(code-V1T2PAQ12)**

(b) $m = \log_{10} 2$ and $n = \log_{10} \sqrt[3]{10}$

(c) $m = \log_{10} 5.\log_{10} 20 + (\log_{10} 2)^2$ and $n = 1$

(d) $m = \log_{1/2}\left(\frac{1}{3}\right)$ and $n = \log_{1/3}\left(\frac{1}{2}\right)$

Que. 2. If $\log_2\left(\log_3\left(\log_4 2^n\right)\right) = 2$ then the value of n can be equal to **(code-V1T10PAQ10)**

(a) $\frac{27}{\log_{27} \tan\left(\frac{4\pi}{3}\right)}$ (b) $\frac{4^{81}}{2}$ (c) $\frac{162}{\log_2 \sec\left(\frac{5\pi}{3}\right)}$ (d) $\frac{81}{\log_4 2}$

Que. 3. The value of x satisfying the equations $2^{2x} - 8.2^x = -12$, is **(code-V1T12PAQ11)**

(a) $1 + \frac{\log 3}{\log 2}$ (b) $\frac{1}{2}\log 6$ (c) $1 + \log\frac{3}{2}$ (d) 1

Que. 4. The expression $\left(\tan^4 x + 2\tan^2 x + 1\right).\cos^2 x$ when $x = \pi/12$ can be equal to **(code-V1T12PAQ13)**

(a) $4\left(2 - \sqrt{3}\right)$ (b) $4\left(\sqrt{2} + 1\right)$ (c) $16\cos^2\frac{\pi}{12}$ (d) $16\sin^2\frac{\pi}{12}$

Que. 5. Given a and b are positive numbers satisfying $4(\log_{10} a)^2 + (\log_2 b)^2 = 1$ then which of the following statement(s) are correct ? **(code-V1T19PAQ21)**

(a) Greatest and least possible values of 'a' are reciprocal of each other.
(b) Greatest and least possible values of 'b' are reciprocal of each other.
(c) Greatest value of 'a' is the square value of 'b'
(d) Least value of 'b' is the square of the least value of 'a'.

Que. 6. Let $\vec{a}, \vec{b}, \vec{c}$ are non zero vectors and $\vec{V}_1 = \vec{a} \times \left(\vec{b} \times \vec{c}\right)$ and $\vec{V}_2 = \left(\vec{a} \times \vec{b}\right) \times \vec{c}$. The vectors $\vec{V}_1$ and $\vec{V}_2$ are equal **(code-V2T9PAQ9)**

(a) $\vec{a}$ and $\vec{b}$ are orthogonal (b) $\vec{a}$ and $\vec{c}$ are collinear

(c) $\vec{b}$ and $\vec{c}$ are orthogonal (d) $\vec{b} = \lambda\left(\vec{a} \times \vec{c}\right)$ when λ is a scalar.

Que. 7. If $\vec{A}, \vec{B}, \vec{C}$ and $\vec{D}$ are four non zero vectors in the same plane no two of which are collinear then which of the following hold(s) good ? **(code-V2T9PAQ10)**

(a) $\left(\vec{A} \times \vec{B}\right).\left(\vec{C} \times \vec{D}\right) = 0$ (b) $\left(\vec{A} \times \vec{C}\right).\left(\vec{B} \times \vec{D}\right) \neq 0$

(c) $\left(\vec{A} \times \vec{B}\right) \times \left(\vec{C} \times \vec{D}\right) = \vec{0}$ (d) $\left(\vec{A} \times \vec{C}\right) \times \left(\vec{B} \times \vec{D}\right) \neq \vec{0}$

Que. 8. Let P_1 denotes the equation of the plane to which the vector $\left(\hat{i}+\hat{j}\right)$ is normal and which contains the line L whose equation is $\vec{r}=\hat{i}+\hat{j}+\hat{k}+\lambda\left(\hat{i}-\hat{j}-\hat{k}\right)$, P_2 denotes the equation of the plane containing the line L and a point with position vector $\hat{j}$. Which of the following holds good ? **(code-V2T9PAQ11)**

(a) equation of P_1 is $x+y=2$

(b) equation of P_2 is $\vec{r}\left(\hat{i}-2\hat{j}+\hat{k}\right)=2$

(c) The acute angle between P_1 and P_2 is $\cot^{-1}\left(\sqrt{3}\right)$

(d) The angle between the plane P_2 and the line L is $\tan^{-1}\sqrt{3}$

Que. 9. If $\vec{a},\vec{b},\vec{c}$ be three non zero vectors satisfying the condition $\vec{a}\times\vec{b}=\vec{c}$ & $\vec{b}\times\vec{c}=\vec{a}$ then which of the following alwyas hold(s) good ? **(code-V2T12PAQ13)**

(a) $\vec{a},\vec{b},\vec{c}$ are orthogonal in pairs

(b) $\left[\vec{a}\ \vec{b}\ \vec{c}\right]=\left|\vec{b}\right|$

(c) $\left[\vec{a}\ \vec{b}\ \vec{c}\right]=\left|\vec{c}\right|^2$

(d) $\left|\vec{b}\right|=\left|\vec{c}\right|^2$

Que. 10. If $\log_a x=b$ for permissible values of a and x then identify the statement(s) which can be correct?

(a) If a and b are two irrational numbers then x can be rational. **(code-V2T15PAQ8)**

(b) If a rational and b irrational then x can be rational

(c) If a irrational and b rational then x can be rational

(d) If a rational and b rational then x can be rational.

Match Matrix Type

Que. 1.

	Column - I		Column - II
A.	$\sin\left(410^\circ-A\right)\cos\left(400^\circ+A\right)+\cos\left(410^\circ-A\right)\sin\left(400^\circ+A\right)$	P.	-1 **(code-V1T2PBQ1)**
B.	$\dfrac{\cos^2 1^\circ-\cos^2 2^\circ}{2\sin 3^\circ.\sin 1^\circ}$ is equal to	Q.	0
C.	$\sin\left(-870^\circ\right)+\operatorname{cosec}\left(-660^\circ\right)+\tan\left(-855^\circ\right)$ $+2\cos\left(840^\circ\right)+\cos\left(480^\circ\right)+\sec\left(900^\circ\right)$	R.	$\dfrac{1}{2}$
D.	If $\cos\theta=\dfrac{4}{5}$ where $\theta\in\left(\dfrac{3\pi}{2},2\pi\right)$ and $\cos\phi=\dfrac{3}{5}$ where $\phi\in\left(0,\dfrac{\pi}{2}\right)$ then $\cos(\theta-\phi)$ has the value equal to	S.	1

Que. 2. **Column - I** **Column - II**

A. Let $f(x)=\left(a^2+a+2\right)x^2-\left(a+4\right)x-7.$ if unity lies between the roots of f(x) = 0 then possible integral value(s) of ‘a’ is/are — P. 1 **(code-V1T4PBQ1)**

B. The possible integral value(s) of x satisfying the inequality $1<\dfrac{3x^2-2x+8}{x^2+1}<2,$ is/are — Q. 2

C. If $\sin x\cos 4x+2\sin^2 2x=1,-4\sin^2\left(\dfrac{\pi}{4}-\dfrac{x}{2}\right)$ then the value(s) of sin x, is/are — R. 4

D. The possible integral value(s) of x satistying theequation $\left(\log_5 x\right)^2+\log_{5x}\left(\dfrac{5}{x}\right)=1,$ is/are — S. 5.

Que. 3. **Column - I** **Column - II**

A. Let ABCDEFGHIJKL be a regular dodecagon. The value of $\dfrac{AB}{AF}+\dfrac{AF}{AB}$ is equal to — **P.** 2 **(code-V1T6PBQ1)**

B. Assume that θ is a rational multiple of π such θ is a distinct rational Number of values of $\cos\theta$ is — **Q.** 3

C. $\dfrac{\log_2 3.\log_4 5.\log_6 7}{\log_4 3.\log_6 5\log_8 7}$ is equal — **R.** 4

D. Number of values of ‘t’ satistying the equation $\cos\left(\sin\left(\cos t\right)\right)=1$ for $t\in[\theta,2\pi]$ — **S.** 5

Que. 4. **Column - I** **Column - II**

A. If x and y are positive real numbers and p,q are any positive integers then the possible value which the xpression $\dfrac{\left(1+x^{2p}\right)\left(1+y^{2q}\right)}{x^p y^q}$ can take is — **P.** 3 **(code-V1T6PBQ2)**

B. Given a and b are positive numbers not equal to1 such that $\log_b\left(a^{\log_2 b}\right)=\log_a\left(b^{\log_4 a}\right)$ and $\log_a\left(p-\left(b-9\right)^2\right)=2$ then the minimum integral value for p is — **Q.** 5

C. Possible values of x simultaneouly satisfying the system of inequalties $\dfrac{(x-6)(x-3)}{x+2}\ge 0$ and $\dfrac{x-5}{x+1}\le 3$ is — **R.** 9

D. If the numbers, $\left(3^{1+x}+3^{1-x}\right),(a/2),\left(9^x+9^{-x}\right)$ form an A.P. , then possible value(s) of ‘a’ is/are $\left(x\in R\right)$ — **S.** 10

Que. 5. **Column - I** **Column - II**

A. $\dfrac{8\sin 40^\circ.\sin 50^\circ.\tan 10^\circ}{\cos 80^\circ}$ equals **P.** 1

B. The value of $\dfrac{1}{\log_2\left(\frac{1}{6}\right)}-\dfrac{1}{\log_3\left(\frac{1}{6}\right)}-\dfrac{1}{\log_4\left(\frac{1}{6}\right)}$ is equal to **Q.** 2

C. $4\log_{18}\left(\sqrt{2}\right)+\dfrac{2}{3}\log_{18}(729)$ is equal to **R.** 3

D. Number of expression(s) which simplifies to $\left(\sec^2\theta\operatorname{cosec}^2\theta\right)$, is are **S.** 4

(i) $\sec^2\theta+\operatorname{cosec}^2\theta$ **(ii)** $(\tan\theta+\cot\theta)^2$ **(code-V1T10PBQ1)**

(iii) $\dfrac{\tan^2\theta+1}{1-\cos^2\theta}$ **(iv)** $\operatorname{cosec}^2 2\theta$

Que. 6. **Column - I** **Column - II**

A. If a, b, c and d are four non zero real numbers such that $(d+a-b)^2+(d+b-c)^2=0$ and the roots of the equation $a(b-c)x^2+b(c-a)x+c(a-b)=0$ are real and equal then **P.** $a+b+c=0$ **(code-V1T14PBQ2)**

B. If a, b, c are real non zero positive numbers such that $\log a, \log b, \log c$ are in A.P. then **Q.** a, b, c are in A.P.

C. If the equation $ax^2+bx+c=0$ and $x^2-3x^2+3x-1=0$ have a common real root then **R.** a, b, c are in G.P.

D. Let a, b, c be positive real numbers such that the expression $bx^2+\left(\sqrt{(a+c)^2+4b^2}\right)x+(a+c)$ is non negative $\forall\ x\in R$ then **S.** a, b, c are in H.P.

Que. 7. **Column - I** **(code-V1T16PBQ1)** **Column - II**

A. Sum of the positive integers which satisty the inequality $\dfrac{x^2-2x-15}{(x^2-3x+4)(x-2)}\le 0$ is **P.** 9

B. The difference between the sum of the first k terms of the $1^3+2^3+3^3+........+n^3$ and the sum of the first k terms of $1+2+3+.........+n$ is 1980. The vlaue of k is **Q.** 10

C. Sides of a triangle ABC are 5, 12, 13. The radius of the escribed circle touching the side of length 12, is **R.** 11

D. If $x^2+x+1=0$ then the value of **S.** 12

$$\left|\left(x+\frac{1}{x}\right)^1+\left(x^2+\frac{1}{x^2}\right)^3+\left(x^4+\frac{1}{x^4}\right)^5+......+\left(x^{1023}+\frac{1}{x^{1024}}\right)\right|$$ equals

Que. 8. **Column - I** **(code-V1T18PBQ1)** **Column - II**

A. The value of 'a' for which the equation
$(\sin x)^{\sqrt{3x-1}} + 2(\cos 2x)^{\sqrt{-9x^2-3x+2}} + \log_{1/3} x^3 = a$
has atleast one solution is — **P.** 2.

B. Let function $f(x) = ax^3 + bx^2 + cx + d$ has 3 positive roots. If the sum of the roots of f(x) is 4, the largest possible integral value of c/a is — **Q.** 4.

C. If A_1 be the A.M. and G_1, G_2 be two G.M.'s between two positive — **R.** 5.
Number 'a' and 'b', then $\dfrac{G_1^3 + G_2^3}{G_1 G_2 A_1}$ is equal to — **S.** 6.

Que. 9. **Column - I** **(code-V1T18PBQ2)** **Column - II**

A. Let $x^2 + y^2 + xy + 1 \geq a(x+y)\ \forall\ x, y \in R$ then the possible integer(s) in the range of a can be — **P.** –1.

B. Let α, β, γ be measures of angle such that $\sin\alpha + \sin\beta + \sin\gamma \geq 2$ then the possible integral values which $\cos\alpha + \cos\beta + \cos\gamma$ can attain — **Q.** 0.

C. The terms a_1, a_2, a_3 form an arithmetic sequence whose sum is 18. — **R.** 1.
The terms $a_1 + 1, a_2, a_3 + 2,$ in that order, form ageometric sequence.
The sum of all possible common difference of the A.P., is

D. The elements in the solution set of $\sqrt{x^2 - 4x + 4 < 3}$ and $\dfrac{1}{4} \leq \dfrac{1}{3-x} \leq \dfrac{1}{2}$, is — **S.** 2.

Que. 10. Match the Statement / Expression in **Column - I** with the Statements /Expressions in **Column - II.**

Column - I **(code-V1T20PBQ2)** **Column - II**

A. If $a^2 - 4a + 1 = 4,$ then the value of $\dfrac{a^3 - a^2 + a - 1}{a^2 - 1}\left(a^2 \neq 1\right)$ is equal to — **P.** 1.

B. The value(s) of x satisfying the equation $\sqrt[4]{|x-3|^{x+1}} = \sqrt[3]{|x-3|^{x-2}}$ is — **Q.** 2.

C. The value(s) of x satisfying the equation $3^x + 1 - \left|3^x - 1\right| = 2\log_5|6-x|$ is — **R.** 4.

D. If the sum of the first 2n terms of the $A.P. 2, 5, 8.............$ is equal to the sum of the first n terms of the $A.P. 57, 59, 61, ...,$ then n equals — **S.** 11.

Que. 11. **Column - I** **(code-V1T20PBQ3)** **Column - II**

A. If the roots of $10x^3 - nx^2 - 54x - 27 = 0$ are in harmonic progression, then 'n' equal — **P.** 4.

B. A preson has 'n' friends. The minimum value of 'n' so that a preson can invite a different pair of friends every day for four weeks in a row is — **Q.** 6.

C. If ${}^{2n+1}P_{n-1} : {}^{2n-1}P_n = 3:5,$ then 'n' equals — **R.** 8.

D. There are two sets of parallel lines, their equation being — **S.** 9.
$x\cos\alpha + y\sin\alpha = p$ and $x\sin\alpha + y\cos\alpha = p;\ p = 1, 2, 3, ..., n$ and
$\alpha \in (0, \pi/2)$. If the number of rectangles formed by these two sets of lines is 225 then the vlaue of n is equal to

Que. 12. **Column - I** **(code-V2T3PBQ2)** **Column - II**

A. The set of values of x satisfying the equation $\int_0^x t^2.\sin(x-t)dt = x^2$, is/are

B. The set of values of 'x' satisfying the equation $\cos 4x + 6 = 7\cos 2x$, is/are

C. The set of values of 'x' for which the expression $\dfrac{\sin\frac{x}{2}+\cos\frac{x}{2}-i\tan x}{1-2i\sin\frac{x}{2}}$ is real, is/are

D. The set of values of 'x' satisfying the equation, $\sin x + \sin 5x = \sin 2x + \sin 4x$ is/are

P. $n\pi\,(n \in I)$

Q. $(4n+1)\dfrac{\pi}{4}(n \in I)$

R. $\dfrac{n\pi}{3}(n \in I)$

S. $2n\pi\,(n \in I)$

Que. 13. **Column I** **(code-V2T6PBQ1)** **Column II**

A. $\lim\limits_{x\to 0}\dfrac{(3\sin x-\sin 3x)^4}{(\sec x-\cos x)}$ is equal to

B. Given that x, y, z are positive reals such that $xyz = 32$. The mininum value of $x^2+4xy+4y^2+2z^2$ is equal

C. The number of ways in which 6 men can be seated so that 3 particular men are consecutive is

D. The number $N = 6^{\log_{10}40}.5^{\log_{10}36}$ is

P. 96

Q. 144

R. 216

S. 256

Que. 14. **Column I** **(code-V2T6PBQ2)** **Column II**

A. Let $f(x) = \dfrac{x^2-3x+2}{x^2+x+6}$. The value of x for which f(x) is equal to $\dfrac{1}{5}$ is

B. Given f(x) is a function such that $f(x) = \begin{cases} x^\alpha \sin\dfrac{1}{x} & \text{if } x > 0 \\ 0 & \text{if } x = 0 \end{cases}$

where α is a constant. If f(x) is a derivable $\forall\, x \ge 0$, then α is

C. Let A and B be 3 × 3 matrices with integer entries, such that $AB = A + B$. The value of det $(A - I)$ can be (where I denotes a 3× 3 unit matrix)

D. If $x = 2008(a-b)$, $y = 2008(b-c)$ and $z = 2008(c-a)$ then the numerical value of $\dfrac{x^2+y^2+z^2}{xy+yz+zx}$ is equal to $'p'(xy+yz+zx \ne 0)$. The value of 'p' so obtained is less than

P. – 1

Q. 1.

R. 2.

S. non existent.

Que. 15. **Column - 1** **(code-V2T7PBQ1)** **Column - II**

A. If the vlaue of $\lim\limits_{x\to0^+}\left(\frac{(3/x)+1}{(3/x)-1}\right)^{1/x}$ can be expressed in the form of $e^{p/q}$, where p and q are relative prime then $(p+q)$ is equal to — **P.** 2.

B. The area of a triangle ABC is equal to $\left(a^2+b^2-c^2\right)$, where a,b and c are the sides of the triangle. The value of tan C equals — **Q.** 3.

C. If the value of y (greater than 1) satisfying the equation $\int\limits_1^y x\,\ell n\,x\,dx=\frac{1}{4}$ can be expressed in the form of $e^{m/n}$, where m and n are relative prime then $(m+n)$ is equal to — **R.** 4.

D. Number of integeral value of x satisfying $\log_2(1+x)=\log_3\left(1+2^x\right)$ — **S.** 5.

Que. 16. **Colum - I** **(code-V2T10PBQ1)** **Column - II**

A. In a ΔABC maximum value of $\cos^2A+\cos^2B+\cos^2C$, is — **P.** 3/4

B. If a , b are c are positive and $9a+3b+c=90$ then the maximum value of $(\log a+\log b+\log c)$ is (base of the logarithm is 10) — **Q.** 2

C. $\lim\limits_{x\to0}\frac{\tan x\sqrt{\tan x}-\sin x\sqrt{\sin x}}{x^2.\sqrt{x}}$ equals — **R.** 3.

D. If $f(x)=\cos\left(x\cos\frac{1}{x}\right)$ and $g(x)=\frac{\ell n\left(\sec^2x\right)}{x\sin x}$ are both continuous at $x=0$ then $f(0)+g(0)$ equals — S. Non existent

Que. 17. **Colum - I** **(code-V2T10PBQ2)** **Column - II**

A. Let $f:R\to R$ and $f_n(x)=f\left(f_{n-1}(x)\right)\forall n\ge2, n\in N$, the roots of equaltion $f_3(x)f_2(x)f(x)-25f_2(x).f(x)+175f(x)=375.$ Which also satisfy equation $f(x)=x$ will be — **P.** 1.

B. Let $f:[5,10]$ onto $[4,17]$, the integers in the range of $y=f\left(f(f(x)\right)$ is/are — **Q.** 5.

C. Let $f(x)=8\cot^{-1}(\cot x)+5\sin^{-1}(\sin x)+4\tan^{-1}(\tan x)-\sin\left(\sin^{-1}x\right)$ then possible integral values which $f(x)$ can take — **R.** 10.

D. Let 'a' denote the roots of equation $\cos\left(\cos^{-1}x\right)+\sin^{-1}\sin\left(\frac{1+x^2}{2}\right)=2\sec^{-1}(\sec x)$ then posible values of $\left[|10a|\right]$ where [.] denotes the greatest integer function will be — **S.** 15.

Que. 18. **Column - I** **(code-V2T9PBQ1)** **Column - II**

A. Let O be an interior point of ΔABC such that $\overrightarrow{OA}+2\overrightarrow{OB}+3\overrightarrow{OC}=\vec{0}$, then the ratio of the area of ΔABC to the area of ΔAOC, is — **P.** 0.

B. Let ABC be a triangle whose centroid is G orthocentre is H and circumcentre is the origin 'O'. If D is any point in the plane of the triangle such that no three of O,A, B, C and D are — **Q.** 1.

C. If $\vec{a},\vec{b},\vec{c}$ and $\vec{d}$ are non zero vectors such that no three of them are in the same plane and no two are orthogonal then the value of the scalar $\dfrac{(\vec{b}\times\vec{c}).(\vec{a}\times\vec{d})+(\vec{c}\times\vec{a}).(\vec{b}\times\vec{d})}{(\vec{a}\times\vec{b}).(\vec{d}\times\vec{c})}$ is — **R.** 2.

S. 3.

Que. 19. **Colum - I** **(code-V2T10PBQ1)** **Column - II**

A. In a ΔABC maximum value of $\cos^2 A+\cos^2 B+\cos^2 C$, is — **P.** 3/4

B. If a , b are c are positive and $9a+3b+c=90$ then the maximum value of $(\log a+\log b+\log c)$ is (base of the logarithm is 10) — **Q.** 2

C. $\lim\limits_{x\to 0}\dfrac{\tan x\sqrt{\tan x}-\sin x\sqrt{\sin x}}{x^2.\sqrt{x}}$ equals — **R.** 3.

D. If $f(x)=\cos\left(x\cos\dfrac{1}{x}\right)$ and $g(x)=\dfrac{\ell n\left(\sec^2 x\right)}{x\sin x}$ are both continuous at $x=0$ then $f(0)+g(0)$ equals — S. Non existent

Que. 20. **Colum - I** **(code-V2T10PBQ2)** **Column - II**

A. Let $f:R\to R$ and $f_n(x)=f\left(f_{n-1}(x)\right)\forall n\geq 2, n\in N$, the roots of equaltion $f_3(x)f_2(x)f(x)-25f_2(x).f(x)+175f(x)=375.$ Which also satisfy equation $f(x)=x$ will be — **P.** 1.

B. Let $f:[5,10]$ onto $[4,17]$, the integers in the range of $y=f\left(f(f(x)\right)$ is/are — **Q.** 5.

C. Let $f(x)=8\cot^{-1}(\cot x)+5\sin^{-1}(\sin x)+4\tan^{-1}(\tan x)-\sin\left(\sin^{-1}x\right)$ then possible integral values which $f(x)$ can take — **R.** 10.

D. Let 'a' denote the roots of equation $\cos\left(\cos^{-1}x\right)+\sin^{-1}\sin\left(\dfrac{1+x^2}{2}\right)=2\sec^{-1}(\sec x)$ then posible values of $\left[|10a|\right]$ where $[.]$ denotes the greatest integer function will be — **S.** 15.

Que. 21. **Column - I** **(code-V2T12PBQ1)** **Column - II**

A. Let a,b are real number such that $a+b=1$ then the minimum value of the integral $\int_0^{\pi}(a\sin x+b\sin 2x)^2\,dx$ is equal to — **P.** $\frac{\pi}{2}$.

B. $\int_0^{\pi/2} x\left|\sin^2 x-\frac{1}{2}\right|dx$ is equal — **Q.** $\frac{\pi}{4}$

C. A rectangle is inscribed in a semicircle with one of its sides along the diameter. If k times the area of the largest rectangle equals the area of the semicircle then the value of k equals — **R.** $\frac{\pi}{8}$

D. If $\vec{a},\vec{b},\vec{c}$ are non-coplanar unit vectors such that $\vec{a}\times(\vec{b}\times\vec{c})=\frac{1}{\sqrt{2}}(\vec{b}+\vec{c})$ then the angle between the vectors $\vec{a},\vec{b}$ is — **S.** $\frac{3\pi}{4}$

Que. 22. **Column - I** **Column - II**

A. Let $f(x)$ is a derivable function satisfying $f(x)=\int_0^x e^t\sin(x-t)dt$ and $g(x)=f''(x)-f(x)$ then thepossible integers **(code-V2T12PBQ2)** in the range of g(x) is — **P.** -1.

B. If the substitution $x=\tan^{-1}(t)$ transforms the differential equation $\frac{d^2y}{dx^2}+xy\frac{dy}{dx}+\sec^2 x=0$ in to adifferential equation $(1+t^2)\frac{d^2y}{dt^2}+(2t+y\tan^{-1}(t))\frac{dy}{dt}=k$ then k is equal to — **Q.** 0.

C. If $a^2+b^2=1$ then (a^3b-ab^3) can be equal to — **R.** 1.

D. If the system of equation — **S.** 2.

$$\left.\begin{array}{l}x-\lambda y-z=0\\ \lambda x-y-z=0\\ x+y-z=0\end{array}\right\}$$

has a unique solution, then the value of λ can be

Que. 23. **Column - I** **(code-V2T15PBQ1)** **Column - II**

A. The sum $\sum_{n=1}^{\infty}\arctan\left(\frac{2}{n^2}\right)$ equals — **P.** $\frac{\pi}{4}$.

B. $\lim_{n\to\infty} n\sin\left(2\pi\sqrt{1+n^2}\right)(n\in N)$ equals — **Q.** $\frac{\pi}{2}$.

C. Period of the function $f(x)=\sin^2 2x+\cos^4 2x+2,$ is — **R.** $\frac{3\pi}{4}$.

D. $\int_{-1}^{1}(1+x)^{1/2}(1-x)^{3/2}dx$ equals — **S.** π.

Que. 24. **Column - I** **(code-V2T16PBQ1)** **Column - II**

A. If the constant term in the binomial expansion of $\left(x^2-\frac{1}{x}\right)^n, n \in N$ is 15 then the vlalue of n is equal to — **P.** 4.

B. The positive value of 'c' that makes the area bounded by the graph of $y=c\left(1-x^2\right)$ and the x-axis equal to 1, can be expressed in the form p/q where $p,q \in N$ and in their lowest form, then $(p+q)$ equals — **Q.** 6.

C. Suppose a, b, c are such that the curve $y=ax^2+bx+c$ is tangent to $y=3x-3$ at (1,0) and is also tangent to $y=x+1$ at (3,4) then the value of $(2a-b-4c)$ equals — **R.** 7.

D. Suppose F_1, F_2 are the foci of the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$. P is a point on ellipse such that $PF_1 : PF_2 = 2:1$. The area of the triangle PF_1F_2 is — **S.** 9.

Que. 25. **Column - I** **(code-V2T16PBQ2)** **Column - II**

A. Let $f(x)=\int x^{\sin x}\left(1+x\cos x.\ell n\, x+\sin x\right)dx$ and $f\left(\frac{\pi}{2}\right)=\frac{\pi^2}{4}$ then the value of $f(\pi)$ is — **P.** rational

B. Let $g(x)=\int \frac{1+2\cos x}{(\cos x+2)^2}dx$ and $g(0)=0$ then the value of $g(\pi/2)$ is — **Q.** Irrational

C. If real numbers x and y satisfy $(x+5)^2+(y-12)^2=(14)^2$ then the minimum value of $\sqrt{\left(x^2+y^2\right)}$ is — **R.** Integral

D. Let $k(x)=\int \frac{\left(x^2+1\right)dx}{\sqrt[3]{x^2+3x+6}}$ and $k(-1)=\frac{1}{\sqrt[3]{2}}$ then the value of $d(-2)$ is — **S.** Prime

Que. 26. **Column - I** **Column - II**

A. If $\log\left(\frac{x^2}{y^3}\right)=1$ and $\log\left(x^2y^2\right)=7$ then $\log\left(|xy|\right)$ is equal to — **P.** 0.

B. $f(x)=\min.\left\{|x|, x^2, 2\right\}, x \in [-5,5]$ number of points where f(x) is not derivable, is — **Q.** 3/2

(code-V2T19PBQ1)

C. $\int_0^2 \frac{2x^3-6x^2+9x-5}{x^2-2x+5}dx$ is equal to — **R.** 3.

D. If the range of the function $f(x)=\log_2\left(4^{x^2}+3^{(x-1)^2}\right)$ is $[a,\infty)$ then the value of 'a' equals — **S.** 4.

Subjective Type (Up to 4 digit)

Que. 1. Find the rational number represented by $\log_{10}\left(\sqrt{3-\sqrt{5}}+\sqrt{3+\sqrt{5}}\right)$. **(code-V1T1PAQ3)**

Que. 2. The number $N=\dfrac{\log_5 250}{\log_{50}5}-\dfrac{\log_5 10}{\log_{1250}5}$, when simplified reduces to a natural number. Find N. **(code-V1T1PAQ5)**

Que. 3. Suppose x, y, z > 0 and different then one and $\ell n\, x+\ell n\, y+\ell n\, z=0$. Find the value of $x^{\frac{1}{\ell n\, y}+\frac{1}{\ell n\, z}}.y^{\frac{1}{\ell n\, z}+\frac{1}{\ell n\, x}}.z^{\frac{1}{\ell n\, x}+\frac{1}{\ell n\, y}}$. **(code-V1T1PAQ6)**

Que. 4. Let L denotes antilog$_{32}$0.6
and M denotes the number of positive integers which have the charactreistics 4, when the base of log is 5 and N denotes the vlaue of $49^{(1-\log_7 2)}+5^{-\log_5 4}$. Find the value of $\dfrac{LM}{N.}$ **(code-V1T1PAQ7)**

Que. 5. Find the solution set of the inequality $2\log_{\frac{1}{4}}(x+5)>\dfrac{9}{4}\log_{\frac{1}{3\sqrt{3}}}(9)+\log_{\sqrt{x+5}}(2)$. **(code-V1T3PAQ6)**

Que. 6. If the inequality $(\log_2 x)^4-\left(\log_{\frac{1}{2}}\dfrac{x^5}{4}\right)-20\log_2 x+148<0$ holds true in (a, b) where $a,b\in N$. Find the value of $ab(a+b)$.

Que. 7. Let $p=\log_5\log_5(3)$. If $3^{C+5^{-p}}=405$, find the value of C. **(code-V1T9PAQ4)**

Que. 8. Let $\log_3 N=\alpha_1+\beta_1$ **(code-V1T9PAQ6)**
$\log_5 N=\alpha_2+\beta_2$
$\log_7 N=\alpha_3+\beta_3$

where $\alpha_1,\alpha_2,\alpha_3$ are integers and $\beta_1,\beta_2,\beta_3\in[0,1)$.

(a) Find the number of integral values of N if $\alpha_1=4$ and $\alpha_2=2$.

(b) Find the largest integral value of N if $\alpha_1=5,\alpha_2=3$ and $\alpha_3=2$.

(c) Find the difference of largest and smallest integral values of N if $\alpha_1=5,\alpha_2=3$ and $\alpha_3=2$.

Que. 9. If $x,y\in R^+$ and $\log_{10}(2x)+\log_{10}y=2$ and $\log_{10}x^2-\log_{10}(2y)=4$, then $x+y=m/n$ where m and n are relatively prime find $(m+n)$.

Que. 10. Solve the inequality : $\sqrt{\log_2\left(\dfrac{2x-3}{x-1}\right)}<1$. **(code-V1T11PAQ4)**

Que. 11. You are designing a 1782cm^3 closed right circular cylindrical cans whose manufacture will take waste into account. There is no waste in cutting the aluminium sheet for the curved surface, but the tops and bottom of radius "r" will be cut from squares that measure "2r" units on a side. Find the total quantity of aluminium (in square cm) for the manufacture of a most economical can.
(code-V2T8PDQ1)

Que. 12. 10 points are taken on one of the two given lines and 20 points on the other (none of these points are common both the line.). Join with lines segments each of the 10 points on the former line to each of the 20 points on the latter. Find the number of points of intersection of the segments. Assume that there are no such points in which three or more segments intersects. **(code-V2T8PDQ2)**

Que. 13. If the lattice point $P(x, y, z), x, y, x \in I$ with the largest value of z such the P lies on the planes $7x+6y+2z=272$ and $x-y+z=16$ (given x, y, z > 0), find the vlaue of $(x+y+z)$. **(code-V2T9PDQ1)**

Que. 14. Given $\vec{A}=2\hat{i}+3\hat{j}+6\hat{k},\ \vec{B}=\hat{i}+\hat{j}-2\hat{k}$ and $\vec{C}=\hat{i}+2\hat{j}+\hat{k}$. Compute the value of $\left|\vec{A}\times\left(\vec{A}\times\left(\vec{A}\times\vec{B}\right)\right).\vec{C}\right|$

(code-V2T9PDQ2)

[SOLUTION]

Single Correct Type

Que. 1. (B)

$N=\log_{10}64+\log_{10}31=\log_{10}1984$

$\therefore \quad 3<N<4 \quad \Rightarrow \quad 7.$

Que. 2. (C)

Answer is $x\in(1,3)\cup(3,5); |x-3|>2 \quad \Rightarrow \quad -2<x-3<2 \quad \Rightarrow \quad 1<x<5, x\neq 3 \quad x\in\{2,4\}$

Que. 3. (A) 1st equation gives $\log_p\left(\log_q\left(\log_r x\right)\right)=0 \Rightarrow \log_q\left(\log_1 x\right)=1$

$\therefore \quad \log_1 x=q \Rightarrow x=r^q$ .... (1)

2nd equation gives $\log_r\left(\log_p x\right)=1 \Rightarrow \log_p x=r \Rightarrow x=p^r$ .... (2)

from (1) and (2) $\quad r^q=p^r \Rightarrow p=r^{q/r}$

Que. 4. **(A)** $\log\left(\frac{1}{7}\right)=P\log\left(\frac{12}{21}\right)+Q\log\left(\frac{11}{22}\right) \quad \Rightarrow \quad \log\left(\frac{1}{7}\right)=P\log\left(\frac{4}{7}\right)+Q\log\left(\frac{1}{2}\right)$

$\log\left(\frac{1}{7}\right)=P\log 4-P\log 7-Q\log 2 \quad \Rightarrow \quad \log\left(\frac{1}{7}\right)=(2P-Q)\log 2-P\log 7$

$P=1;\quad 2P-Q=0 \quad \Rightarrow \quad Q=2 \quad \Rightarrow \quad 7P-Q=5.$

Que. 5. **(C)** $\left(\log_a x^2\right)\log_a x=(k-2)\log_a x-k$ (taking log on base a)

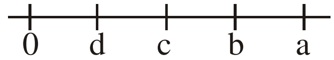
let $\quad \log_a x=t \qquad 2t^2-(k-2)t+k=0;$

put $\quad D=0$ (has only one solution) $\quad (k-2)^2-8k=0 \Rightarrow k^2-12k+4=0 \Rightarrow$ sum $=12.$

Que. 6. (a) $\log_2 x\left[1+\frac{1}{2}+\frac{1}{3}\right]=\log_2 x.\log_k 2;\ \frac{11}{6}\log_2 x=\log_2 x.\log_k 2;\ \log_2 x\left[\frac{11}{6}-\log_k 2\right]=0$

$\therefore \ \log_k 2\frac{11}{6};\log_2 k=\frac{6}{11}\Rightarrow k=2^{\frac{6}{11}}=(64)^{\frac{1}{11}}=a^{\frac{1}{b}} \therefore\ a=64$ and $b=11\Rightarrow a+b=64+11=75.$

Que. 7. (b) $(ab-cd)^2=(a^2+d^2)(b^2+c^2)-(ac+bd)^2=1-\frac{1}{9}=\frac{8}{9}$



as $ab-cd\ge 1\Rightarrow ab-cd=\frac{2\sqrt{2}}{3}$.

Alternatively : put $a=\sin\theta; d=\cos\theta; b\sin\phi; c\cos\phi$ $(ab-cd\ge 0)$ also given that $\sin\theta\cos\phi+\sin\phi\cos\theta$

$=-\cos(\theta+\phi)\Rightarrow\frac{1}{3}\Rightarrow\sin(\theta+\phi)=\frac{1}{3}\therefore\ \cos^2(\theta+\phi)=1-\frac{1}{9}=\frac{8}{9}\Rightarrow\cos(\theta+\phi)=\frac{2\sqrt{2}}{3}$.

Que. 8. (d) $\frac{1}{\log_a x}+\frac{1}{\log_a y}=\frac{4}{\log_a x+\log_a y};\frac{\log_a y+\log_a x}{\log_a x.\log_a y}=\frac{4}{\log_a x+\log_a y}$

$(\log_a x+\log_a y)^2=4\log_a x.\log_a y=(\log_a x-\log_a y)^2=0\Rightarrow\log_a x=\log_a y\Rightarrow x=y.$

Que. 9. B. Given $2\log T+\log O+\log K=2$ .....(1), $2\log K+\log O+\log E=3$ .....(2),

and $2\log E+\log T+\log O=4$ .....(3) add $\log(T^3E^3C^3O^3)=9\Rightarrow\log(TECK)=3\Rightarrow TEKO=1000.$

Que. 10. A. $10^p\le P<10^{p+1}\Rightarrow P=10^{p+1}-10^p;\ P=9.10^p$

$|||\ell y$ $10^{q-1}<Q\le 10^q\Rightarrow Q=10^2-10^{2-1}=10^{2-1}(10-1)=9.10^{q-1}$

$\therefore\ \log_{10}P\log_{10}Q=\log_{10}(P/Q)=\log_{10}10^{p-q+1}=p-q+1.$

Que. 11. A. $\vec{a}-\vec{p}+\vec{b}=\vec{p}+\vec{c}-\vec{p}=0$ $\Rightarrow$ $\vec{p}=\frac{\vec{a}+\vec{b}+\vec{c}}{3}$ $\Rightarrow$ **A.**

Que. 12. A. Both the lines pass through origin Line L_1 is parallel to the vector

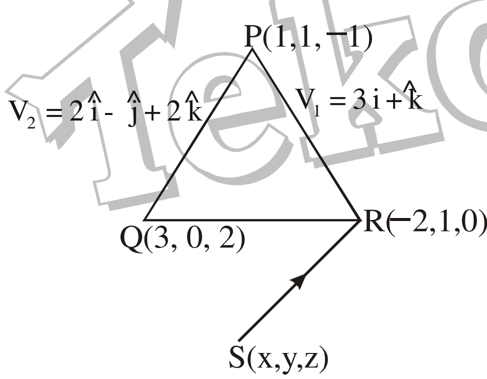
$\overrightarrow{V_1}=(\cos\theta+\sqrt{3})\hat{i}+(\sqrt{2}\sin\theta)\hat{j}+(\cos\theta-\sqrt{3})\hat{k}$ and L_2 is parallel to the vector. $\overrightarrow{V_2}=a\hat{i}+b\hat{j}+c\hat{k}$

$$\therefore\ \cos\alpha=\frac{\overrightarrow{V_1}.\overrightarrow{V_2}}{|\overrightarrow{V_1}||\overrightarrow{V_2}|}=\frac{a(\cos\theta+\sqrt{3})+(b\sqrt{2})\sin\theta+c(\cos\theta-\sqrt{3})}{\sqrt{a^2+b^2+c^2}\sqrt{(\cos\theta+\sqrt{3})^2+2\sin^2\theta+(\cos\theta-\sqrt{3})^2}}$$

$=\frac{(a+c)\cos\theta+b\sqrt{2}\sin\theta+(a-c)\sqrt{3}}{\sqrt{a^2+b^2+c^2}\sqrt{2+6}}$ in order that $\cos\alpha$ is independent of θ $a+c=0$ and $b=0$

$\therefore$ $\cos\alpha=\frac{2a\sqrt{3}}{a\sqrt{2}.2\sqrt{2}}=\frac{\sqrt{3}}{2}$ $\Rightarrow$ $\alpha=\frac{\pi}{6}$.

Que. 13. D. $\overrightarrow{V_1},\overrightarrow{V_2},\overrightarrow{RS}$ are in the same plane $\therefore$ $(2\hat{i}-\hat{j}+3\hat{k})\times(-3\hat{i}+\hat{k}).((x+2)\hat{i}+(y-1)\hat{j}+2\hat{k})=0$ actual palne is $x+11y+3z=6$.

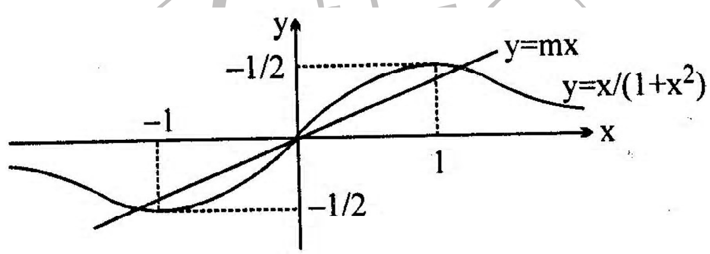


Que. 14. C. $\left|\vec{u}\times\vec{v}\right|^2 \vec{u}^2\vec{v}^2-\left(\vec{u}.\vec{v}\right)^2 \Rightarrow 36=(9)(4)-\left(\vec{u}.\vec{v}\right)^2 \Rightarrow \vec{u}.\vec{v}=0 \Rightarrow$ $\vec{u}$ and $\vec{v}$ are orthogonal also $\left(\vec{u}\times\vec{v}\right)\times\vec{u}=\left(\vec{u}.\vec{v}\right)\vec{v}-\left(\vec{v}\vec{u}.\right)\vec{u}=9\vec{v}$ $\Rightarrow$ **(D)** is incorrect.

Que. 15. C. Obviously (C); $R\left(\vec{r}\right)$ moves on PQ

$$P(\vec{P}) \quad R(\vec{r}) \quad Q(\vec{q})$$

Que. 16. B. Solving $mx=\frac{x}{x^2+1}\Rightarrow x^2+1=\frac{1}{m}$ or $x=0\Rightarrow x^2=\frac{1}{m}-1>0$ for a region $\frac{m-1}{m}<0$

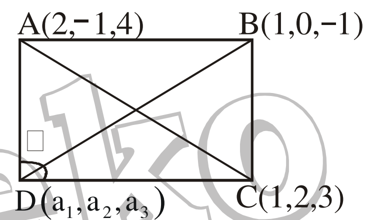


$\Rightarrow m\in(0,1)$ **Note :** form $=0$ or 1 the line does not enclose a region.

Comprehesion Type

1 Paragraph for Q. 1 to Q. 3

1. **C.** **2.** **B.** **3.** **D.**



(i)
$a_1+1=3 \Rightarrow a_1=1$
$a_2+0=1 \Rightarrow a_2=1$
$a_3-1=7 \Rightarrow a_3=8$

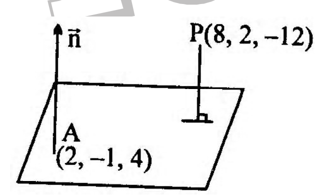
$\vec{d}=\left|\frac{\left(\overrightarrow{AB}\right)\times\left(\overrightarrow{AD}\right)}{\left|\overrightarrow{AB}\right|}\right|$; $\overrightarrow{AB}=\hat{i}-\hat{j}+5\hat{k}$; $\overrightarrow{AD}=0\hat{i}+2\hat{j}+4\hat{k} \Rightarrow \overrightarrow{AB}\times\overrightarrow{AD}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -1 & 5\\ 0 & 2 & 4\end{vmatrix}$

$=(-4-10)\hat{i}-(4)\hat{j}+(2)\hat{k}=-4\hat{i}-4\hat{j}+2\hat{k}=-2\left(7\hat{i}+2\hat{j}-\hat{k}\right)=2\sqrt{2}.$

(ii) $\vec{n}=7\vec{i}+2\vec{j}-\vec{k}$ is normal to plane $\therefore$ $P=(8,2,-12) \Rightarrow \overrightarrow{AP}=6\hat{i}+3\hat{j}-16\hat{k}$

$\therefore$ Distance $d=\left|\frac{\overrightarrow{AP}.\vec{n}}{\left|\vec{n}\right|}\right|=\left|\frac{42+6+16}{\sqrt{49+4+1}}\right|=\frac{64}{\sqrt{54}}=\frac{64}{3\sqrt{6}}=\frac{64\sqrt{6}}{18}=\frac{32\sqrt{6}}{9}.$ Figer.

(iii) Vector normal to the plane in RHS $\overrightarrow{AD}\times\overrightarrow{AB}=+2\left(7\hat{i}+2\hat{j}-\hat{k}\right)$ projection of xy = 2; $yz=14$; $zx=4$



 # 2 Paragraph for Q. 4 to Q. 6

4. **A.** **5.** **D.** **6.** B .

$a+b-c=2$ ....(1) and $2ab-c^2=4$ ....(2)

$a^2+b^2+c^2+2ab-2bc-2ca=4=2ab-c^2 \Rightarrow (b-c)^2+(a-c)^2=0 \Rightarrow a=b=c$ (triangle is equiliateral)

triangle is equilateral also $a=2$ form (1) $\Rightarrow$ area ofo $\Delta ABC=\frac{\sqrt{3}}{4}.4=\sqrt{3}$

Also $f(x)=2(x^2+x+1) \Rightarrow$ f is increasing in [0,1] $\Rightarrow$ $f(x)|_{max}=6$

Hcence answer of (i) is (A); (ii) is (D) and (iii) is (B) $\left(r_1=\frac{\Delta}{s-a}=\sqrt{3}\right)$

Assertion & Reason Type

Que. 1. **(B).**

Que. 2 **(A)**

Que. 3. (D) $\sqrt{\log_x \cos(2\pi x)}$ is a meaningful quantity only if $x\in(0,1/4)\cup(3/4,1)$ and $x=2,3,4,5$.........

Que. 4. (d) $\log_{10} N=-20\log_{10}(4/10)=20[1-2\log_{10}2]=20[1-2\times 0.301]=20\times 0.301=7.96$

$\therefore$ Number of digits = characteristic $+1=7+1=8$ $\therefore$ statement -1 is false.

Que. 5. B. $x^3+1-x^2=1 \Rightarrow x^2(x-1)=0 \Rightarrow x=\{0,1\}$ both satisty. Statement - 2 can not be the correct explanation as even with Statement - 2 correct the equation $\Rightarrow$ $36k=9[(507)(11)] \Rightarrow K=1394.25$. has no real solution.]

Que. 6. **D.** S – 1 is false. In (0,1) inequality does not satisfy.

$\Rightarrow$ Statement - 2 is true but this can not be take as the correct explanation.

Que. 7. **D.** L_1 and L_2 are obviously not parallel Consider the determinant

$$D=\begin{vmatrix}2 & -4 & 1\\ 2 & 4 & -3\\ 1 & 3 & 2\end{vmatrix}=2(8+9)+4(4+3)+1(6-4)=34+28+2 \Rightarrow D\neq 0 \Rightarrow \text{skew hence S-1 is flase.}$$

Que. 8. **A.** Think ! obvious.

More than One May Correct Type

Que. 1. (A,D)

(A) $m-n=(\log_2 5)^2-[\log_2 5+2]$

let $\log_2 5=x=x^2-x-2=(x-2)(x+1)=(\log_2 5-2)(\log_2 5+1)>0$ Hence $m>n$ $\Rightarrow$ (A) is Ans.

(B) $m=\log_{10}2=0.3010;$ $n=\frac{1}{3}=0.333$........; hence $n>m$

(C) $m=(1-\log_{10}2)(1+\log_{10}2)=1-(\log_{10}2)^2$ and $n=1$.

$\therefore$ $m-n=-(\log_{10}2)^2<0$

$\therefore$ $m<n$

(D) $m=\log_2 3;$ $n=\log_3 2$ $\Rightarrow$ $m>n$ (D) is Ans.

Que. 2. (A,C,D) $\log_3\left(\log_4 2^n\right)=4;\ \log_4 2^n=81;\ 4^{81}-2^n;\ n=162$ now verify each alternative.

Que. 3. A,D. $2^x=t\Rightarrow t^2-8t+12=0\Rightarrow(t-6)(t-2)=0\ \Rightarrow 2^x=6\Rightarrow x=\log_2 6=1+\dfrac{\log 3}{\log 2}\Rightarrow$ (A)

$\rightarrow\quad 2^x=2\quad\Rightarrow\quad 1\quad\Rightarrow\quad$ (D)

Que. 4. A,D. $y=\dfrac{\left(1+\tan^2 x\right)^2}{1+\tan^2 x}=1+\tan^2=\sec^2 x=\left(\dfrac{4}{\sqrt{6}+\sqrt{2}}\right)^2=16.\dfrac{\left(\sqrt{6}-\sqrt{2}\right)^2}{16}=8-4\sqrt{3}=4\left(2-\sqrt{3}\right).$

$=4\left[\left(\sqrt{\dfrac{3}{2}}-\dfrac{1}{\sqrt{2}}\right)^2\right]=4\left[4\left(\dfrac{\sqrt{3}-1}{2\sqrt{2}}\right)^2\right]=16.\sin^2\dfrac{\pi}{12}.$

Que. 5. A,B. $(\log_2 b)^2=1-(2\log_{10}a)^2\geq 0\ \Rightarrow(2\log_{10}a)^2-1\leq 0\ \Rightarrow\ (2\log_{10}a-1)(2\log_{10}a+1)\leq 0$

$\Rightarrow\ \log_{10}a\in\left[-\dfrac{1}{2},\dfrac{1}{2}\right];\ \ a\in\left[\dfrac{1}{\sqrt{10}},\sqrt{10}\right]$ $\quad$ |||ℓy $\ (\log_{10}a)^2=\dfrac{1-(\log_{10}b)^2}{4}\geq 0\ \Rightarrow\ (\log_{10}b)^2-1\leq 0$

$\Rightarrow\ (\log_{10}b-1)(\log_{10}b+1)\leq 1\ \Rightarrow\ \log_{10}b\in[-1,1]\ \therefore\ b\in\left[\dfrac{1}{10},10\right].$

Que. 6. B,D. $\vec{V}_1=\vec{V}_2\ \Rightarrow\ \vec{a}\times\left(\vec{b}\times\vec{c}\right)=\left(\vec{a}\times\vec{b}\right)\times\vec{c}\ \Rightarrow\ \left(\vec{a}.\vec{c}\right)\vec{b}-\left(\vec{a}.\vec{b}\right)\vec{c}=\left(\vec{a}.\vec{c}\right)\vec{b}-\left(\vec{b}.\vec{c}\right)\vec{a}\quad\therefore\ \left(\vec{a}.\vec{b}\right)\vec{c}=\left(\vec{b}.\vec{c}\right)\vec{a}$

$\Rightarrow$ ether $\vec{c}$ and $\vec{a}$ are collinear or $\vec{b}$ is perpendicular to both $\vec{a}$ and $\vec{c}$ $\ \Rightarrow\ \vec{b}=\lambda\left(\vec{a}\times\vec{c}\right).$

Que. 7. B,C. Obviously (B) and (C).

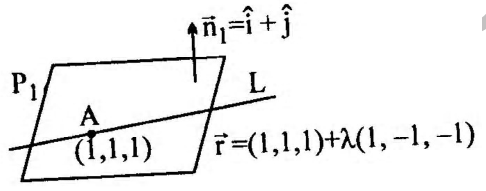
Que. 8. A,C. $\left(\vec{r}-\vec{a}\right).\vec{n}_1=0\ \Rightarrow\ \vec{r}.\vec{n}_1=\vec{a}.\vec{n}\ \Rightarrow\ \left(x\hat{i}+y\hat{j}+z\hat{k}\right).\left(\hat{i}+\hat{j}\right)=2\ \Rightarrow\ x+y=2\ \Rightarrow$ **(A)** is correct.

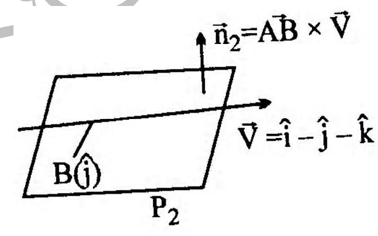
Now $\ \overrightarrow{AB}=\hat{i}+\hat{k}$ Now $\ \overrightarrow{AB}=\vec{V}+\vec{n}_2=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&0&1\\1&-1&-1\end{vmatrix}$ $\ \vec{n}_2=\hat{i}(0+1)-\hat{j}(-1-1)+\hat{k}(-1)=\vec{n}_2=\hat{i}+2\hat{j}-\hat{k}$

Hence equation of P_2 is $\left(\vec{r}-\hat{j}\right).\left(\hat{i}+2\hat{j}-\hat{k}\right)=0\ \Rightarrow\ \vec{r}.\left(\hat{i}+2\hat{j}-\hat{k}\right)=2\ \Rightarrow$ **(B)** is not correct.

If θ is the acute angle between P_1 and P_2 then $\cos\theta=\dfrac{\vec{n}_1.\vec{n}_2}{\left|\vec{n}_1\right|\left|\vec{n}_2\right|}=\left|\dfrac{\left(\hat{i}+\hat{j}\right).\left(\hat{i}+2\hat{j}-\hat{k}\right)}{\sqrt{2}.\sqrt{6}}\right|=\dfrac{3}{\sqrt{2}.\sqrt{6}}=\dfrac{\sqrt{3}}{2}$

$\theta=\cot^{-1}\sqrt{3}=\dfrac{\pi}{6}\quad\Rightarrow\quad$ **(C)** is correct. As L is contained in $P_2\ \Rightarrow\ \theta=0.$





Que. 9. **A,C.** Clearly $\vec{a}.\vec{c}=0\ \&\ \vec{b}.\vec{c}=0$ Also $\vec{a}.\vec{b}=0 \Rightarrow$ A.

Again $\begin{matrix} |\vec{a}||\vec{b}|=|\vec{c}| \\ |\vec{b}||\vec{c}|=|\vec{a}| \end{matrix} \Rightarrow \frac{|\vec{a}|}{|\vec{c}|}=\frac{|\vec{c}|}{|\vec{c}|} \Rightarrow |\vec{a}|=|\vec{c}|\ \&\ |\vec{b}|=1 \ \Rightarrow \vec{a}\times\vec{b}.\vec{c}=|\vec{a}||\vec{b}||\vec{c}|=|\vec{a}|^2=|\vec{c}|^2$ (children will assume $\vec{a}=\hat{i};\vec{b}=\hat{j}$ and $\vec{c}=\hat{k}$ but in this case all the four will be correct which will be wrong)

Que. 10. **A,B,C,D.** (A). $a=\left(\sqrt{2}\right)^{\sqrt{2}}$ is irrational, $b=\sqrt{2}$ is alos irrational

(B). $a=2\in Q;\ b=\log_2 3\notin Q$

$a^b=2^{\log_2 3}=3\notin Q \quad \Rightarrow \quad$ **(B)** is correct.

But $a^b=\left(\sqrt{2}^{\sqrt{2}}\right)^{\sqrt{2}}$ which is rational $\Rightarrow$ **(A)** is correct.

Match Matrix Type

Que. 1. **[A - S. B - R. C - P. D - Q.]**

(A) $\text{SIN}\left(410^\circ+400^\circ\right)=\sin 810^\circ=\sin\left(720^\circ+90^\circ\right)=\sin 90^\circ=1 \Rightarrow$ (S)

(B) $\frac{\sin^2 2^\circ-\sin^2 1^\circ}{2\sin 3^\circ\sin 1^\circ}=\frac{\sin 3^\circ \sin 1^\circ}{2\sin 3^\circ\sin 1^\circ}=\frac{1}{2} \Rightarrow$ (R)

(C) $-\sin(810^\circ+60^\circ)-\text{cosec}\left(720^\circ-60^\circ\right)-\tan\left(810^\circ+45^\circ\right)+2\cos 120^\circ+\cos 120^\circ+\sec 180^\circ$

$=-\frac{1}{2}+\frac{2}{\sqrt{3}}+1-\frac{2}{\sqrt{3}}-\frac{1}{2}-1=-1 \Rightarrow$ (P)

(D) $\cos\left(\theta-\phi\right)=\cos\theta\cos\phi+\sin\theta\sin\phi=\frac{4}{5}\times\frac{3}{5}-\frac{3}{5}\times\frac{4}{5}=0 \Rightarrow$ (Q)

Que. 2. **A - P,Q.** **B - Q,R,S.** **C - P.** **D - P,S.**

A. coeffcient of $x^2>0$ (always) $f(1)<0$ $\left(a^2+a+2\right)-a-4-7<0$ $a^2-9<0 \Rightarrow -3<a<3$

$\Rightarrow$ non zero integral values of $'a'$ are $\{-2,-1,1,2\} \Rightarrow$ (P),(Q)

B. $x^2+1<3x^2-7x+8$ $2x^2-7x+7>0$ $D<0 \Rightarrow$ always true

again $3x^2-7x+8<2x^2+2$ $x^2-7x+6<0$ $(x-6)(x-1)<0 \Rightarrow x\in(1,6)$

$\therefore$ $x\in\{2,3,4,5\} \Rightarrow$ (Q),(R),(S),

C. $\sin x.\cos 4x=\cos 4x-2\left[1-\cos\left(\frac{\pi}{2}-x\right)\right]=\cos 4x-2+2\sin x$ $\quad \left(1-2\sin^2 x=\cos 4x\right)$

$\therefore$ $\cos 4x\left(1-\sin x\right)-2\left(1-\sin x\right)=0$ $\quad \left(1-\sin x\right)\left(\cos 4x-2\right)=0$

$\cos 4x\neq 2 \Rightarrow \sin x=1 \Rightarrow$ (P)

D. Put $\log_5 x=y$ $\left(\log_5 x\right)^2+\log_{5x}5-\log_5 x=1 \Rightarrow \left(\log_5 x\right)^2+\frac{1}{1+\log_5 x}-\frac{1}{\log_x 5+1}=1$

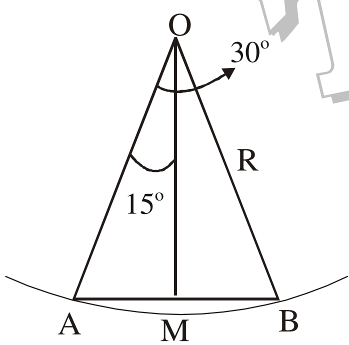
$y^2+\frac{1}{1+y}-\frac{y}{1+y}=1 \quad \frac{1-y}{1+y}+y^2-1=0$

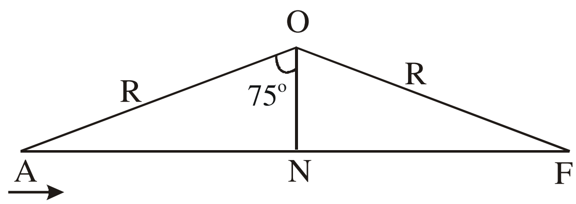
$(y-1)\left[(y+1)-\frac{1}{y+1}\right]=0 \quad y=1 \text{ or } (y+1)^2-1=0 \quad y=1 \text{ or } y=0 \text{ or } y=-2$

$x=5;\ \ x=1;\ \ x=\frac{1}{25} \quad \Rightarrow \quad (P),(S)$

Que. 3. A - R. B - S. C - Q. D - P.

A. $AM=R\sin\frac{\pi}{12}\Rightarrow AB=2R\sin\frac{\pi}{12}$ |||ℓy $AN=R\sin\frac{5\pi}{12}\Rightarrow AF=2R\sin\frac{5\pi}{12}$





$\therefore \quad \frac{AB}{AF}+\frac{AF}{AB}=\frac{\sin\frac{\pi}{12}}{\sin\frac{5\pi}{12}}+\frac{\sin\frac{5\pi}{12}}{\sin\frac{\pi}{12}}=\tan\frac{\pi}{12}+\cot\frac{\pi}{12}=\left(2-\sqrt{3}\right)+\left(2-\sqrt{3}\right)=4.$

B. $\theta=k\pi, k=\frac{p}{q}.p,q\in I, q\neq 0$ cos $k\pi$ is a rational hence $k=0,1,1/2,1/3,2/3\Rightarrow 5$ value of $\cos\theta$

for which $\cos\theta$ is rational i.e. $\cos\theta\in\{\pm1,0,\pm1/2\}\Rightarrow 5\Rightarrow S.$

C. $\frac{\log_2 3.\log_4 5.\log_6 7}{\log_4 3.\log_6 5.\log_8 7}=\frac{\log 3}{\log 2}.\frac{\log 5}{\log 4}.\frac{\log 7}{\log 6}.\frac{\log 4}{\log 3}.\frac{\log 6}{\log 5}.\frac{\log 8}{\log 7}=\frac{\log 8}{\log 2}=\log_2 8=3 \quad\Rightarrow\quad Q$

D. $\cos(\sin(\cot))=1\Rightarrow \sin(\cos t)=2n\pi, n\in I \quad \therefore \ \cos t=0\Rightarrow t\in\frac{\pi}{2} \text{ or } \frac{3\pi}{2}\Rightarrow$ number of values of 't' are 2. $\Rightarrow$ P.

Que. 4. A - Q,R,S. B - R. C - P,R,S. D - R,S.

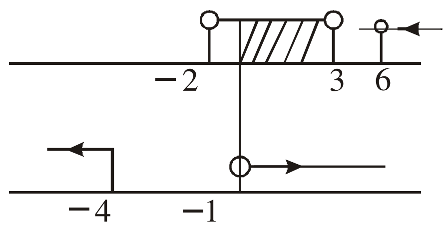
A. $\geq 4 \quad\Rightarrow\quad Q,R,S$

B. Given $\log_b\left(a^{\log_2 b}\right)=\log_a\left(b^{\log_4 a}\right)$ or $\log_2 b.\log_b a=\log_4 a.\log_a b\Rightarrow 2\log_2 b.\log_b a=\log_2 a.\log_a b$

$\Rightarrow 2\log_2 a=\log_2 b\Rightarrow a^2=b$ now $p-(b-9)^2=a^2=b\Rightarrow p-\left(b^2+81+18b\right)=b\Rightarrow p=b^2+81-17b$

$=\left(b-\frac{17}{2}\right)^2+81-\frac{289}{4}=\left(b-\frac{17}{2}\right)+\frac{35}{4};$ Hence $p_{min}=\frac{35}{4}=8.75\Rightarrow$ minimum integral value $=9$

C. $\frac{(x-6)(x-3)}{x+2} \geq 0 \Rightarrow \frac{x-5}{x+1} - 3 \leq 0 \Rightarrow \frac{x-5-3x-3}{x+1} \leq 0 \Rightarrow \frac{-2x-8}{x+1} \leq 0 \Rightarrow \frac{x+4}{x+1} \geq 0$



$\Rightarrow$ common solution is $(-1,3] \cup [6,\infty) \Rightarrow$ P, R, S.

D. $a = 3^{1+x} + 3^{1-x} + 9^x + 5^x \Rightarrow 3(3^x + 3^{-x}) + (9^x + 9^{-x}) \Rightarrow \geq 6+3 \Rightarrow \geq 9 \Rightarrow$ R, S.

Que. 5. A - S. **B -** P. **C -** Q. **D-** R.

A. $\frac{4(2\sin 50^o.\sin 40^o)\sin 10^o}{\sin 10^o.\cos 10^o} = \frac{4(\cos 10^o - \cos 90^o)\sin 10^o}{\sin 10^o.\cos 10^o} = 4 \Rightarrow$ S.

B. $\log_{1/6}(2) - \log_{1/6}(3) - \log_{1/6}(4) = \log_{1/6}\left(\frac{2}{3.4}\right) = \log_{1/6}\left(\frac{1}{6}\right) = 1 \Rightarrow$ P.

C. $2\log_{18} 2 + \frac{2}{3}.6\log_{18} 3;\ \log_{18} 4 + \log_{18} 81 = \log_{18}(324) = 2 \Rightarrow$ Q.

D. **(i)** $\sec^2\theta + \operatorname{cosec}^2\theta = \frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta} = \frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta\sin^2\theta} = \frac{1}{\cos^2\theta}.\frac{1}{\sin^2\theta} = \sec^2\theta + \operatorname{cosec}^2\theta$

(ii) $(\tan\theta + \cot\theta)^2 = \left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)^2 = (\sec\theta\operatorname{cosec}\theta)^2 = \sec^2\theta\operatorname{cosec}^2\theta$

(iii) $\frac{\tan^2\theta + 1}{1-\cos^2\theta} = \frac{\sec^2\theta}{\sin^2\theta} = \sec^2\theta.\frac{1}{\sin^2\theta} = \sec^2\theta\operatorname{cosec}^2\theta$

(iv) $\frac{1}{\sin^2 2\theta} = \frac{1}{4\sin^2\theta\cos^2\theta} = \frac{\sec^2\theta\operatorname{cosec}^2\theta}{4}$ hence number of expression = 3 $\Rightarrow$ R.

Que. 6. A - Q,R,S. B - R. C - P. D - Q.

A. $d + a - b = 0$ and $d + b - c = 0 \Rightarrow d = b - a$ and $d = c - b$

$\therefore\ b - a = c - b \Rightarrow 2b = a + b \Rightarrow$ a, b, c are in A.P. $\Rightarrow$ (Q)

also x = 1 satisfies the 2nd equation $\therefore$ other root is also 1 $\therefore$ product of roots = 1

$\therefore\ c(a-b) = a(b-c) \Rightarrow b = \frac{2ac}{a+c} \Rightarrow$ a, b, c are in H.P. $\therefore$ a, b, c are in A.P. and a,b,c in H.P .

$\Rightarrow$ a, b, c in G.P. $\Rightarrow$ (R), (S)

B. $2\log b = \log a + \log c = b^2 = ac \Rightarrow$ (R)

C. $(x-1)^3 = 0 \Rightarrow x = 1$ is common root hence a + b + c = 0 $\Rightarrow$ (P)

D. $(a+c)^2 + 4b^2 - 4b(a+c) \leq 0\ (D<0) \Rightarrow ((a+c)-2b)^2 \leq 0 \Rightarrow a+c = 2b \Rightarrow$ a, b, c in A.P. $\Rightarrow$ (Q).

Que. 7. A - S. B - P. C - Q. D - R.

A. $x \in \{5,4,3\} \Rightarrow 12$

B. $\left[\frac{k(k+1)}{2}\right]^2 - \frac{k(k+1)}{2} = 1980 \Rightarrow \frac{k(k+1)}{2}\left[\frac{k(k+1)}{2} - 1\right] = 1980$

$k(k+1)(k^2+k-2) = 1980 \times 4 \Rightarrow (k-1)k(k+2) = 8.9.10.11 \therefore k = -1 = 8 \Rightarrow k = 9.$

C. $\Delta = 30 \Rightarrow s = 15;\ r = \frac{\Delta}{s-a} \Rightarrow r = \frac{30}{15-12} = 10.$

D. $x^2 + x + 1 = 0 \Rightarrow x + \frac{1}{x} = -1 \Rightarrow x^4 + \frac{1}{x^4+2} = 1 \Rightarrow x^4 + \frac{1}{x^4} = -1$ etc.

there are 11 term $\therefore \quad |-1-1-1-........11 \text{ times}| = 11.$

Que. 8. A - S. B - R. C - P.

A. $3x - 1 \ge 0$ and $-9x^2 - 3x + 3 \ge 0$ or $(3x-1)(3x+2) \le 0$

$\Rightarrow \quad x = 1/3$ corresponding to which $a = 6.$

B. Let $ax^3 + bx^2 + cx + d = 0 <_{r}^{p\,q} \Rightarrow pq + qr + rp = \frac{c}{a}$ .... (1)

but $pq + qr + rp \le p^2 + q^2 + r^2 = (p+q+r)^2 - 2\sum pq \quad \therefore \quad 3(pq+qr+rp) \le (p+q+r)^2 = 16$

$\therefore \quad 3\frac{c}{a} \le 16 \Rightarrow \frac{c}{a} \le \frac{16}{3} \Rightarrow$ largest possible integral value of $\frac{c}{a}$ is 5.

C. $A_1 = \frac{a+b}{2}, G_1 = a.\left(\frac{b}{a}\right)^{1/3}, G_2 = a.\left(\frac{b}{a}\right)^{2/3} \Rightarrow G_1^3 = a^2b,\ G_2^3 = b^2a,\ G_1G_2 = a^2\left(\frac{b}{a}\right) = ab.$

$\Rightarrow \quad \frac{G_1^3 + G_2^3}{G_1G_2A_1} = \frac{ab(a+b).2}{ab.(a+b)} = 2.$

Que. 9. A - P,Q,R. B - P,Q,R,S. C - P. D - Q,R.

A. $x^2 + x(y-a) + y^2 - ay + a \ge 0\ \forall\ x \in R \Rightarrow (y-a)^2 - 4(y^2 - ay + 1) \le 0 \Rightarrow -3y^2 = 2ay + a^2 - 4 \le 0$

$\therefore\ 3y^2 + 2ay + 4 - a^2 \ge 0\ \forall\ y \in R \Rightarrow D \le 0 \Rightarrow 4a^2 - 4.3(4-a^2) \ge 0 \Rightarrow a^2 - 3(4-a^2) \le 0 \Rightarrow 4a^2 - 12 \le 0$

$\therefore$ range of $a \in \left[-\sqrt{3}, \sqrt{3}\right] \Rightarrow$ Number of integer $\{-1,0,1\}$.

Que. 10. A - S. B - R. C - Q. D - P.

A. Given $a^2 - 4a + 1 = 4 \Rightarrow a^2 + 1 = 4(1+a) \Rightarrow y = \frac{(a-1)(1+a^2)}{a^2-1} = \frac{a^2+1}{a+1} = \frac{4(a+1)}{a+1} = 4.$

B. $\sqrt[4]{|x-3|^{x+1}} = \sqrt[3]{|x-3|^{x-2}}$ taking log on both the sides $\frac{x+1}{4}\log|x-3| = \frac{x-2}{3}\log|x-3|$

$\Rightarrow \log|x-3|\left[\left(\frac{x+1}{4}\right) - \left(\frac{x-2}{3}\right)\right] = 0 \Rightarrow \log|x-3| = 0$ or $\left[\left(\frac{x+1}{4}\right) - \left(\frac{x-2}{3}\right)\right] = 0$

$\Rightarrow \quad x = 4, 2 \quad$ or $\quad x = 11.$

C. Critical points $x = 0, 6$

Case I: $\quad x < 6 \Rightarrow 3^x + 1 - (3^x - 1) = 2\log_5(6 - x) \Rightarrow x = 11.$

Case II: $\quad 0 \le x \le 6 \Rightarrow 3^x + 1 - (3^x - 1) = 2\log_5(6 - x) \Rightarrow x = 1.$

Case III: $\quad x < 0 \Rightarrow 3^x + 1 + 3^x - 1 = 2\log_5(6 - x) \Rightarrow 3^x = \log_5(6 - x)$

for $x < 0$ L.H.S. is less than one and R.H.S. is greater than one $\Rightarrow$ one solution.

D. $\frac{2n}{2}(4 + (2n - 1)3) = \frac{n}{2}(114 + (n - 1)2) \Rightarrow 2(1 + 6n) = 112 + 2n \Rightarrow 110 = 10n \Rightarrow n = 11.$

Que. 11. A - S. B - R. C - P. D - Q.

A. $10x^3 - nx^2 - 54x - 27 = 0$ roots in H.P. put $x = 1/t \Rightarrow 27t^3 + 54t^2 + nt - 10 = 0 \begin{cases} a-d \\ a \\ a+d \end{cases}$ root in A.P. $\therefore \quad 3a = -\frac{54}{27} \Rightarrow a = -\frac{2}{3} \qquad (a - d)a(a + d) = \frac{10}{27} \qquad -\frac{2}{3}\left(\frac{4}{9}d^2\right) = \frac{10}{27} \Rightarrow \left(\frac{4}{9} - d^2\right) = -\frac{5}{9}$

$\therefore \quad d^2 = 1 \Rightarrow d = \pm 1$ with $d = -1 \Rightarrow -\frac{2}{3} + 1, -\frac{2}{3}, -\frac{2}{3} - 1 \Rightarrow \frac{1}{3}, -\frac{2}{3}, -\frac{5}{3}$ with $d = 1$

$\Rightarrow -\frac{2}{3} - 1, -\frac{2}{3}, -\frac{2}{3} + 1 \Rightarrow -\frac{5}{3}, -\frac{2}{3}, \frac{1}{3} \Rightarrow \frac{n}{27} = \frac{10}{9} - \frac{5}{9} - \frac{2}{9} \Rightarrow \frac{n}{27} = \frac{3}{9} \Rightarrow n = 9.$

B. ${}^nC_2 = 28 \Rightarrow n = 8$ (7 days a week).

C. $\frac{{}^{2n+1}P_{n-1}}{{}^{2n-1}P_n} = \frac{3}{5}; \quad \frac{{}^9P_3}{{}^7P_4} = \frac{9!}{6!} \times \frac{3!}{7!} = \frac{3}{5}$ verify with $n = 4.$

D. ${}^nC_2 . {}^nC_2 = 225 \Rightarrow n = 6.$

Que. 12. A - S. B - P,S. C - Q,S. D - P,R,S.

A. $I\int_0^x \underbrace{t^2}_{I} . \underbrace{\sin(x - t)}_{II} dt = x^2$ integrating by parts

$= t^2 . \cos(x - t)\Big]_0^x - 2\int_0^x \underbrace{t}_{I} . \underbrace{\cos(x - t)}_{II} dt = x^2 \qquad = (x^2 - 0) = 2\left[-t\sin(x - t)\Big]_0^x + \int_0^x \sin(x - t)dt\right] = x^2$

$= -\left[0 + \cos(x - t)_0^x\right] = 0; \qquad 1 - \cos x = 0 \quad \Rightarrow \quad x = 2n\pi, n \in I. \Rightarrow$ **S.**

B. $2\cos^2 2x - 1 + 6 = 7\cos 2x$ let $\cos 2x = t \Rightarrow 2t^2 - 7t + 5 = 0 \Rightarrow (t - 1)(2t - 5) = 0$

$\cos 2x = 1 \Rightarrow 2x = 2n\pi \Rightarrow n\pi \Rightarrow x = 2n\pi$ will also satisfy. $\Rightarrow$ **P,S.**

C. $z = \frac{\left(\sin\frac{x}{2} + \cos\frac{x}{2} - i\tan x\right)\left(I + 2i\sin\frac{x}{2}\right)}{1 + 4\sin^2\frac{x}{2}}$ should be real $\Rightarrow \quad \mathrm{Im}(z) = 0$

$2\sin\frac{x}{2}\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)-\tan x=0 \quad\Rightarrow\quad 1-\cos x+\sin x-\tan x=0 \Rightarrow \quad 1-\frac{\sin x}{\cos x}+\sin x-\cos x=0$

$\frac{\cos x-\sin x}{\cos x}+(\sin x-\cos x)=0 \;\Rightarrow\quad (\sin x-\cos x)\left(1-\frac{1}{\cos x}=0\right)\Rightarrow\quad (\sin x-\cos x)(\cos x-1)=0$

$\therefore\quad \tan x=1 \quad$ or $\quad \cos x=1 \;\Rightarrow\; n\pi+\frac{\pi}{4};\; x=2n\pi \;\Rightarrow x=(4n+1)\frac{\pi}{4} \quad$ or $\quad 2n\pi. \;\Rightarrow\quad$ **Q,S.**

D. $\quad 2\sin 3x\cos 2x=2\sin 3x\cos x \;\Rightarrow\quad \sin 3x(\cos 2x-\cos x)=0 \quad\Rightarrow\quad \sin 3x=0$

$x=\frac{n\pi}{3}\Rightarrow\quad \cos 2x=\cos x \;\Rightarrow\quad 2x=2n\pi\pm x \quad\Rightarrow\quad x=2n\pi;\quad x=\frac{2n\pi}{3} \quad\Rightarrow\quad$ **P, R, S.**

Que. 13. A -S. B - P. C - Q. D - R.

A. $\quad \lim_{x\to 0}\frac{\left(4\sin^3 x\right)^4}{\left(1-\cos^2 x\right)^6}=\lim_{x\to 0}256\frac{\left(\frac{\sin^3 x}{x^3}\right)^4 .x^{12}}{\left(1-\cos^2 x\right)^6}=256. \quad\Rightarrow\quad$ **S.**

B. $\quad$ Use $AM\ge GM$ for $x^2, 2xy, 2xy, 4y^2, z^2, z^2$

$\therefore\quad \frac{x^2+2xy+2xy+4y^2+z^2+z^2}{6}\ge\left[16(xyz)^4\right]^{1/6}=\left[16(32)^4\right]^{1/4}=(2^{24})^{1/6}=16 \quad\Rightarrow E_{min}=96.$

C. $\quad 4!.3!=144$

D. $\quad \log_{10}N=\log_{10}40.\log 6+\log_{10}36.\log 5=\log_{10}6\left[\log_{10}40+\log_{10}25\right]=\log_{10}6\left[\log_{10}1000\right]$

$-\log_{10}(6)^3 \qquad\therefore\qquad N=6^3=216.$

Que. 14. A - S. B - R. C - P,Q. D - P,Q,R.

A. $\quad \frac{1}{5}=\frac{x^2-3x+2}{x^2+x-6} \quad\Rightarrow\; 5x^2-15x+10=x^2+x-6 \;\Rightarrow\; 4x^2-16x+16=0$

$x^2-4x+4=0 \;\Rightarrow\; x=2$ but for $x=2, f(x)=\frac{0}{0}\Rightarrow$ non existent.

B. $\quad f'(0^+)=\lim_{h\to 0}\frac{f(0+h)-f(0)}{h}=\lim_{h\to 0}\frac{h^{\alpha}\sin(1/h)}{h}=\lim_{h\to 0}h^{\alpha-1}.\sin\frac{1}{h}$ for this limit to exist

$\alpha-1>0 \quad\Rightarrow\quad \alpha>1 \;\Rightarrow\quad \alpha=2.$

C. $\quad$ Given $AB=A+B \Rightarrow\; AB-A-B+I=I$, When I is the unit marix $A(B-I)-(B-I)=I$,

$(A-I)(B-I)=I \quad\Rightarrow\quad \det(A-I)\det(B-I)=1$ since the matrices have integer entries hence det $(A-I)$ and det $(B-I)$ are integer $\therefore$ $\det(A-I)=\det(B-I)=1$ or $\det(A-I)=\det(B-I)=-I$

if $A=B=0 \quad\Rightarrow\quad \det(A-I)=-1 \quad$ if $A=B=2I \quad\Rightarrow\quad \det(A-I)=1.$

D. $\quad$ Let $k=2008,\quad x=k(a-b), y=k(b-c)$ and $z=k(c-a)$

$\therefore\; x+y+z=0 \;\Rightarrow\; x^2+y^2+z^2=-2(xy+yz+zx) \quad\therefore\quad \frac{x^2+y^2+z^2}{xy+yz+zx}=-2 \;\Rightarrow\; p=-2.$

Que. 15. A - S. B - R. C - Q. D - P.

A. $\ell = \lim_{x\to 0}\left(\frac{3+x}{3-x}\right)^{1/x} = e^{\lim_{x\to 0}\frac{1}{x}\left(\frac{3+x}{3-x}-1\right)} = e^{\lim_{x\to 0}\frac{3x}{x(3-x)}} = e^{2/3} \quad \Rightarrow \quad 2+3=5.$

B. $\cos C = \frac{a^2+b^2-c^2}{2ab}$ now $\Delta = a^2+b^2-c^2$ hence $\cos C = \frac{\Delta}{2ab}$ .... (1)

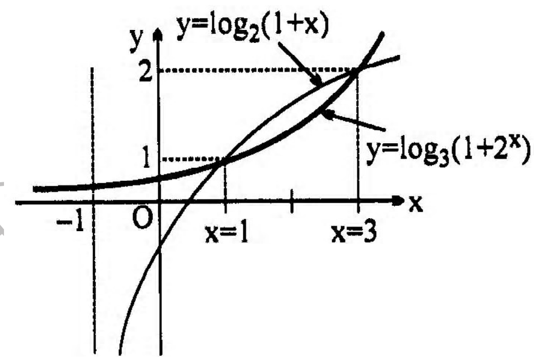
also $\Delta = \frac{1}{2}ab\sin C \Rightarrow \frac{2\Delta}{\sin C} = ab \Rightarrow \sin C = \frac{2\Delta}{ab}$ .... (2) form (1) and (2)

$\tan C - \frac{2\Delta}{ab}, \frac{2ab}{\Delta} = 3.$

C. $\int_1^y x\,\ell n\, x\, dx = \frac{y^2}{2}\ell n\, y - \frac{1}{4}y^2 + \frac{1}{4} \qquad \left[\ell n\, x.\frac{x^2}{2}\Big|_1^y - \frac{1}{2}\int_1^y x\,dx\right]$

$\therefore \frac{y^2}{2}\ell n\, y - \frac{1}{4}y^2 = 0;\ y^2\left[\frac{\ell n\, y}{2} - \frac{1}{4}\right] = 0 \qquad \Rightarrow y = e^{1/2} \Rightarrow 1+2+3.$

D. $x = 1$ or $x = 3 \Rightarrow 2$ solution.



Que. 16. A. - S. B - R. C - P. D - Q.

B. $9a+3b+c = 90^o \Rightarrow 3a+b+\frac{c}{3} = 30$ now using $GM \le AM$ for 3 numbers 3a, b and $\frac{c}{3}$

$\left(3a.b.\frac{c}{3}\right)^{\frac{1}{3}} \le \frac{3a+b+\frac{c}{3}}{3} \quad \Rightarrow \quad (abc)^{1/3} \le \frac{30}{30} = 10 \quad \therefore \quad abc \le 1000$

$\Rightarrow \log a + \log b + \log c \le 3 \Rightarrow \log a + \log b + \log c\big|_{max} = 3. \quad \Rightarrow$ **(R)**

C. $\lim_{x\to 0}\frac{(\tan x)^{3/2}\left[1-(\cos x)^{3/2}\right]}{x^{3/2}.x^2} = 1.\lim_{x\to 0}\frac{1-\cos^2 x}{x^2}.\frac{1}{1+(\cos x)^{3/2}} = \frac{1}{2}.\frac{1}{2}\left(1+\cos x+\cos^2 x\right) = \frac{3}{4}. \Rightarrow$ **(P)**

D. $f(0)+g(0) = 1+1 = 2 \quad \Rightarrow$ **(Q).**

Que. 17. A. - Q,S. B. - Q,R,S. C - P,Q,R,S. D - P,R.

A. $f_2(x) = f(f(x)) = f(x) = x \quad \Rightarrow \quad f_3(x) = f(f_2(x) = f(x) = x \Rightarrow \quad x^3 - 25x^2 + 175x - 375 = 0$

$(x-5)(x^2-20x+75) \Rightarrow (x-5)^2(x-15) = 0 \Rightarrow x = 5,15 \quad \Rightarrow$ **Q,S.**

B. Range of $f\left(f\left(f(x)\right)\right)$ is $\left[4,17\right]$ $\Rightarrow$ **Q,R,S**

Domain of $f(x)$ is $[-1,1]$

C. If $x\in[-1,0),\ f(x)=8(x+\pi)+5x+4x-x=16x+8\pi \Rightarrow f(x)\in[8\pi-16,8\pi]$

If $x\in(0,1],\ f(x)=8x+5x+4x-x=16x \Rightarrow f(x)\in(0,16]$ $\Rightarrow$ **P,Q,R,S.**

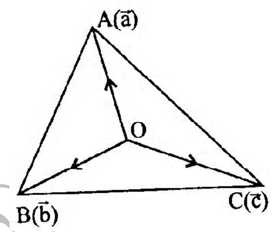
D. $x\in\left[-1,0\right] \Rightarrow x+\dfrac{1+x^2}{2}=-2x \Rightarrow x^2+6X+1=0 \Rightarrow X=\dfrac{-6\pm\sqrt{36-4}}{2}=-3\pm2\sqrt{2}$

$x=2\sqrt{2}-3 \Rightarrow |10a|=\left[\left|20\sqrt{2}-30\right|\right]=30-20\sqrt{20}\,| \Rightarrow x\left[0,1\right] \Rightarrow x+\dfrac{1+x^2}{2}=2x$

$1+x^2=2x \Rightarrow x=1 \Rightarrow |10a|=10 \Rightarrow |10\,a|=10;,\ \left|20\sqrt{2}-30\right| \Rightarrow \left[|10a|\right]=1,10 \Rightarrow$ **P, R.**

Que. 18. A. - S. B - R. C - Q.

A. $\dfrac{\text{Area of }\Delta ABC}{\text{Area of }\Delta AOC}=\dfrac{\frac{1}{2}\left|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}\right|}{\frac{1}{2}\left|\vec{a}\times\vec{c}\right|}$ now $\vec{a}+2\vec{b}+3\vec{c}=0$



cross with $\vec{b},\ \vec{a}\times\vec{b}+3\vec{c}\times\vec{b}=0 \Rightarrow \vec{a}\times\vec{b}=3\left(\vec{b}\times\vec{c}\right)$

cross with $\vec{a},\ 2\vec{a}\times\vec{b}+3\vec{a}\times\vec{c}=0 \Rightarrow \vec{a}\times\vec{b}=\dfrac{3}{2}\left(\vec{c}\times\vec{a}\right)$

$\therefore\ \vec{a}\times\vec{b}=\dfrac{3}{2}\left(\vec{c}\times\vec{a}\right)=3\left(\vec{b}\times\vec{c}\right)$ Let $\left(\vec{c}\times\vec{a}\right)=\vec{p} \Rightarrow \vec{a}\times\vec{b}=\dfrac{3\vec{p}}{2};\quad \vec{b}\times\vec{c}=\dfrac{\vec{p}}{2}$

$\therefore$ ratio $=\dfrac{\left|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}\right|}{\left|\vec{c}\times\vec{a}\right|}=\dfrac{\left|\frac{3\vec{p}}{2}+\frac{\vec{p}}{2}+\vec{p}\right|}{\left|\vec{p}\right|}=\dfrac{3\left|\vec{p}\right|}{\left|\vec{p}\right|}=3. \Rightarrow$ **(S).**

B. $\text{LHS}=\vec{d}-\vec{a}+\vec{d}-\vec{b}+\vec{h}-\vec{c}+3\left(\vec{g}-\vec{h}\right)$ $=2\vec{d}-\left(\vec{a}+\vec{b}+\vec{c}\right)+3\dfrac{\left(\vec{a}+\vec{b}+\vec{c}\right)}{3}-2\vec{h}$

$=2\vec{d}-2\vec{h}=2\left(\vec{d}-\vec{h}\right)=2\overline{HD} \Rightarrow \lambda=2. \Rightarrow$ **(R).**

C. $\left(\vec{b}\times\vec{c}\right).\left(\vec{a}\times\vec{d}\right)=\begin{vmatrix}\vec{b}.\vec{a} & \vec{b}.\vec{d}\\ \vec{c}.\vec{a} & \vec{c}.\vec{d}\end{vmatrix}$ |||ℓy compute others which gives (1) $\Rightarrow$ **(Q).**

Que. 19. A. - S. B - R. C - P. D - Q.

B. $9a+3b+c=90^{\circ} \Rightarrow 3a+b+\dfrac{c}{3}=30$ now using $GM\le AM$ for 3 numbers 3a, b and $\dfrac{c}{3}$

$$\left(3a.b.\frac{c}{3}\right)^{\frac{1}{3}} \le \frac{3a+b+\frac{c}{3}}{3} \quad \Rightarrow \quad (abc)^{1/3} \le \frac{30}{30} = 10 \quad \therefore \quad abc \le 1000$$

$$\Rightarrow \log a + \log b + \log c \le 3 \Rightarrow \log a + \log b + \log c\big|_{max} = 3. \quad \Rightarrow \quad \textbf{(R)}$$

C. $$\lim_{x\to 0}\frac{(\tan x)^{3/2}\left[1-(\cos x)^{3/2}\right]}{x^{3/2}.x^2} = 1.\lim_{x\to 0}\frac{1-\cos^2 x}{x^2}.\frac{1}{1+(\cos x)^{3/2}} = \frac{1}{2}.\frac{1}{2}\left(1+\cos x+\cos^2 x\right) = \frac{3}{4}. \Rightarrow \textbf{(P)}$$

D. $f(0)+g(0) = 1+1 = 2 \quad \Rightarrow \quad$ **(Q).**

Que. 20. A. - Q,S. B. - Q,R,S. C - P,Q,R,S. D - P,R.

A. $f_2(x) = f(f(x)) = f(x) = x \quad \Rightarrow \quad f_3(x) = f(f_2(x) = f(x) = x \Rightarrow \quad x^3 - 25x^2 + 175x - 375 = 0$

$(x-5)(x^2-20x+75) \;\Rightarrow\; (x-5)^2(x-15) = 0 \Rightarrow x = 5,15 \quad \Rightarrow$ **Q,S.**

B. Range of $f(f(f(x)))$ is $[4,17] \quad \Rightarrow$ **Q,R,S**

Domain of $f(x)$ is $[-1,1]$

C. If $x \in [-1,0)$, $f(x) = 8(x+\pi)+5x+4x-x = 16x+8\pi \Rightarrow f(x) \in [8\pi-16, 8\pi]$

If $x \in (0,1]$, $f(x) = 8x+5x+4x-x = 16x \;\Rightarrow\; f(x) \in (0,16] \quad \Rightarrow$ **P,Q,R,S.**

D. $x \in [-1,0] \;\Rightarrow\; x + \frac{1+x^2}{2} = -2x \;\Rightarrow\; x^2+6X+1 = 0 \Rightarrow X = \frac{-6\pm\sqrt{36-4}}{2} = -3\pm 2\sqrt{2}$

$x = 2\sqrt{2}-3 \Rightarrow |10a| = \left[\left|20\sqrt{2}-30\right|\right] = 30-20\sqrt{20}\,] \Rightarrow \; x\,[0,1] \Rightarrow x + \frac{1+x^2}{2} = 2x$

$1+x^2 = 2x \Rightarrow x = 1 \Rightarrow |10a| = 10 \Rightarrow |10\,a| = 10;, \left|20\sqrt{2}-30\right| \Rightarrow \left[|10\,a|\right] = 1,10 \;\Rightarrow$ **P, R.**

Que. 21. A - Q. B - R. C - P. D - S.

A. Let $I = \int_0^{\pi}(a\sin x + b\sin 2x)^2\,dx \;\Rightarrow\; I\int_0^{\pi}(a\sin x - b\sin 2x)^2\,dx$ (using King)

add $\cancel{2}I + \cancel{2}\int_0^{\pi}\left(a^2\sin^2 x + b^2\sin^2 2x\right)dx \;\Rightarrow\; I = 2\int_0^{\pi/2}\left(a^2\sin^2 x\right)dx + 2\int_0^{\pi/2}\left(b^2\sin^2 2x\right)dx$ [Using Queen]

$= 2a^2\frac{\pi}{4} + 2b^2\underbrace{\int_0^{\pi/2}\sin^2 2x\;dx}_{J}$ Let $J = \int_0^{\pi/2}\sin^2 2x\,dx$; Put $2x = t = \frac{1}{2}\int_0^{\pi}\sin^2 t\,dt = \int_0^{\pi/2}\sin^2 t\,dt = \frac{\pi}{4}$

hence $I = \frac{\pi a^2}{2} + \frac{\pi b^2}{2} = \frac{\pi}{2}\left(a^2+b^2\right)$

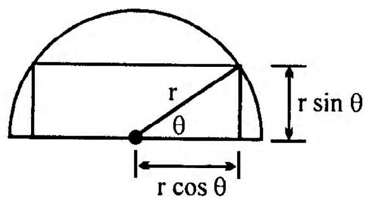
$$\Rightarrow I(a) = \frac{\pi}{2}\left[a^2+(1-a)^2\right] = \frac{\pi}{2}\left[2a^2-2a+1\right] = \pi\left[a^2-a+\frac{1}{2}\right] = \pi\left[\left(a-\frac{1}{2}\right)^2+\frac{1}{4}\right]$$

$\therefore$ I(a) is minimum when $a = \frac{1}{2}$ and minimum value $= \frac{\pi}{4} \quad \Rightarrow$ **(Q).**

B. $I = \frac{1}{2}\int_0^{\pi/2} x|\cos t|dt;\ 2x = t \Rightarrow dx = \frac{dt}{2} \Rightarrow I = \frac{1}{8}\int_0^{\pi} t|\cos t|dt \Rightarrow I = \frac{1}{8}\int_0^{\pi}(\pi - t)|\cos t|dt$

$2I = \frac{\pi}{8}\int_0^{\pi}|\cos t|dt = \frac{2\pi}{8} \Rightarrow I = \frac{\pi}{8}.$ $\Rightarrow$ **(R).**

C. ARea of rectangle $= 2r\cos\theta.r\sin\theta = r^2\sin 2\theta \Rightarrow A_{max.} = r^2 \therefore k.r^2 = \frac{\pi r^2}{2} \Rightarrow k = \frac{\pi}{2}.$ $\Rightarrow$ **(P).**



D. $(\hat{a}.\hat{c})\hat{b} - (\hat{a}.\hat{b})\hat{c} = \frac{1}{\sqrt{2}}\hat{b} + \frac{1}{\sqrt{2}}\hat{c} \therefore \hat{a}.\hat{c} = \frac{1}{\sqrt{2}}$ and $\hat{a}.\hat{b} = -\frac{1}{\sqrt{2}} \Rightarrow \hat{a}\wedge\hat{c} = \frac{\pi}{4};\ \hat{a}\wedge\hat{b} = \frac{3\pi}{4}$ $\Rightarrow$ **(S)**

Que. 22. A - P,Q,R. B - P. C - Q. D - Q,S.

A. $f(x) = \int_0^x e^t\sin(x-t)dt = \int_0^x e^{x-t}\sin(t)dt$ (using King) $\Rightarrow f(x) = e^x\int_0^{\pi} e^{-t}\sin t\,dt$

$f'(x) = e^x.e^{-x}\sin x + \left(\int_0^{\pi} e^{-t}\sin t\,dt\right)e^x \Rightarrow f'(x) = \sin x + f(x)$ .....(1)

$\Rightarrow f''(x) = \cos x + f'(x) = \cos x + \sin x + f(x)$ [Using (1)] $f''(x) - f(x) = \sin x + \cos x$ .....(2)

$\Rightarrow g(x) = \sin x + \cos x \Rightarrow g(x) \in \left[-\sqrt{2}, \sqrt{2}\right] \Rightarrow$ **P,Q,R.**

B. $x = \tan^{-1} t \Rightarrow \frac{dx}{dt} = \frac{1}{1+t^2} \Rightarrow \frac{dy}{dx} = \frac{dy}{dt}.\frac{dt}{dx} = \frac{dy}{dt}(1+t^2)$ ....(1)

$\frac{d^2y}{dx^2} = \frac{d}{dt}\left[\frac{dy}{dt}(1+t^2)\right].\frac{dt}{dx} = \left[\frac{dy}{dt}2t + (1+t^2)\frac{d^2y}{dt^2}\right](1+t^2)$ .... (2) hence the given differential equation $\frac{d^2y}{dx^2} + xy\frac{dy}{dx} + \sec^2 x > 0,$ becomes

$(1+t^2)\left[2t\frac{dy}{dt} + (1+t^2)\frac{d^2y}{dt^2}\right] + y\tan^{-1}t\left[\frac{dy}{dx}(1+t^2)\right] + (1+t^2) = 0$ cancelling $(1+t^2)$ throughout we get.

$(1+t^2)\frac{d^2y}{dt^2} + (2t + y\tan^{-1}t)\frac{dy}{dt} = -1 \Rightarrow k = -1.$ $\Rightarrow$ **P.**

C. Let $a = \cos\theta;\ b = \sin\theta \therefore E = ab(a^2 - b^2) = \cos\theta\sin\theta(\cos 2\theta) = \frac{1}{2}\sin 2\theta\cos 2\theta = \frac{1}{4}\sin 2\theta$

$\Rightarrow -\frac{1}{4} \le E \le \frac{1}{4};$ Possible value $= 0$ $\Rightarrow$ **Q.**

D. obviously $D_1 = D_2 = D_3 = 0 \quad \Rightarrow \quad D = \begin{vmatrix} 1 & -\lambda & -1 \\ \lambda & -1 & -1 \\ 1 & 1 & -1 \end{vmatrix} \neq 0 \; = \begin{vmatrix} 0 & -\lambda & -1 \\ \lambda-1 & -1 & -1 \\ 0 & 1 & -1 \end{vmatrix}$

$= -(\lambda-1)[-1-\lambda] = (\lambda-1)(\lambda+1) \neq 0 \quad \Rightarrow \quad \lambda \neq 1, -1$ hence $\lambda = R - \{-1, 1\} \Rightarrow$ **Q,S.**

Que. 23. A - R. B - S. C - P. D - Q.

A. $T_n = \tan^{-1}\left(\frac{2}{n^2}\right) = \tan^{-1}\left(\frac{2}{1+\left(n^2-1\right)}\right) = \tan^{-1}\left(\frac{2}{1+(n-1)(n+1)}\right) = \tan^{-1}\left(\frac{(n+1)-(n-1)}{1+(n-1)(n+1)}\right)$

$= \tan^{-1}(n+1) - \tan^{-1}(n-1) \Rightarrow T_1 = \tan^{-1}(2) = \tan^{-1}(0) \Rightarrow T_2 = \tan^{-1}(3) - \tan^{-1}(1) \Rightarrow T_3 = \tan^{-1}(4) - \tan^{-1}(2)$

$T_{n-1} = \tan^{-1}(n) - \tan^{-1}(n-2) \Rightarrow T_n = \tan^{-1}(n+1) - \tan^{-1}(n-1) \Rightarrow S = \pi - \tan^{-1}(1) = \frac{3\pi}{4}.$

B. $\ell = \lim_{n\to\infty} n \sin\left(2\pi\sqrt{1+n^2} - 2n\pi\right) = \lim_{n\to\infty} n \sin\left(\frac{2\pi\left(\sqrt{1+n^2}-n\right)}{\left(\sqrt{1+n^2}+n\right)}\right) = \lim_{n\to\infty} \frac{2n\pi}{n\left(\sqrt{1+\frac{1}{n^2}}+1\right)} = \frac{2\pi}{2} = \pi.$

C. $\sin^2 2x + \left(1-\sin^2 2x\right)^2 + 2 \Rightarrow \sin^2 2x + 1 + \sin^4 2x - 2\sin^2 2x + 2$

$\Rightarrow 3 + \sin^4 2x - \sin^2 2x = 3 - \sin^2 2x(1-\sin^2 2x) \Rightarrow 2 - \sin^2 2x.\cos^2 2x \Rightarrow 3 - \frac{\sin^2 4x}{4} \Rightarrow$ period is $\frac{\pi}{4}.$

D. Using property $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx \Rightarrow I = \int_{-1}^{1} (1-x)^{1/2}(1+x)^{3/2} dx$

$2I\int_{-1}^{1} (1+x)^{1/2}(1-x)^{1/2}\left[(1-x)+(1+x)\right]dx \Rightarrow 2I = 2\int_{-1}^{1}\sqrt{1-x^2}dx$

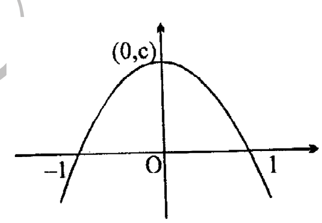
$I = 2\int_0^1 \sqrt{1-x^2}dx \qquad (x = \sin\theta) \Rightarrow dx = \cos\theta\, d\theta \Rightarrow I = 2\int_0^{\pi/2} \cos^2\theta d\theta = \frac{\pi}{2}.$

Que. 24. A - Q. B - R. C - S. D - P.

A. T_{r+1} in $\left(x^2 - \frac{1}{x}\right)^n$ is ${}^nC_r(x^2)^{n-r}(-1)^r x^{-r} = {}^nC_r x^{2n-3r}(-1)^r$ Constant term $= {}^nC_r(-1)^r$ if

$= {}^nC_r(-1)^r$ i.e. coefficient of x = 0 hence ${}^nC_{2n/3}(-1)^{2n/3} = 15 = {}^6C_4 \Rightarrow n = 6.$

B. $2\int_0^1 c\left(1-x^2\right)dx = 1 \Rightarrow 2c\left(1-\frac{1}{3}\right) = 1 \Rightarrow 2c.\frac{2}{3} = 1 \Rightarrow c = \frac{3}{4}$



C. $y = ax^2 + bx + c; \; \frac{dy}{dx} = 2ax + b$ when $x = 1, y = 0 \Rightarrow a + b + c = 0$ ......(1)

$\left.\frac{dy}{dx}\right|_{x=1} = 3$ and $\left.\frac{dy}{dx}\right|_{x=3} = 1 \Rightarrow 2a+b=3$ ....(2) $\Rightarrow 6a+b=1$ .....(3) solving (1), (2) and (3)

$a = -\frac{1}{2};\ b = 4, c = -\frac{7}{2}\ \therefore\ 2a-b-4c = -1-4+14 = 9.$

D. $e^2 = 1-\frac{b^2}{a^2} = 1-\frac{4}{9} = \frac{5}{9} \Rightarrow e = \frac{\sqrt{5}}{3}$

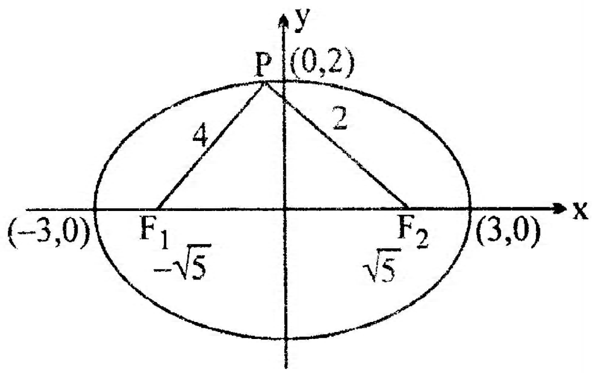
$F_1F_2 = 2\sqrt{5}$ also $PF_1 + PF_2 = 6$

and $PF_1 = 2(PF_2)$ (given)

$\therefore 3PF_2 = 6 \Rightarrow PF_2 = 2$ and $PF_1 = 4$

since $(PF_2)^2 + (PF_1)^2 = (F_1F_2)^2 \Rightarrow \angle P = 90^o$

Area $= \frac{4.2}{2} = 4.$



Que. 25. A - Q. B - P. C - P,R. D - P,R,S.

A. $f(x) = \int x^{\sin x}(1 + x\cos x.\ell n\, x + \sin x)dx$ if $F(x) = x^{\sin x} = e^{\sin x\, \ell n\, x}$ $\therefore$ $xF'(x) = x^{\sin x}(x\cos x\, \ell n\, x + \sin x)$

$\therefore\ f(x)\int(F(x) + xF'(x)) = xF(x) + C \Rightarrow f(x) = x.x^{\sin x} + C \Rightarrow f\left(\frac{\pi}{2}\right) = \frac{\pi}{2}.\frac{\pi}{2} + C \Rightarrow C = 0.$

$\therefore\ f(x) = x(x)^{\sin x};\ f(\pi) = \pi(\pi)^0 = \pi$(irrational).

B. $g(x) = \int \frac{\cos x(\cos x+2)+\sin^2 x}{(\cos x+2)^2}dx = \int \underbrace{\cos x}_{II}.\underbrace{\frac{1}{(\cos x+2)}}_{I}dx + \int \frac{\sin^2 x}{\cos x+2}dx = \frac{1}{\cos x+2}.\sin x$

$-\int \frac{\sin^2 x}{(\cos x+2)^2}dx + \int \frac{\sin^2 x}{(\cos x+2)^2}dx \Rightarrow g(x) = \frac{\sin x}{\cos x+2} + C$

<u>Alternatively</u> : Consider $\frac{d}{dx}\left(\frac{\sin x}{\cos x+2}\right) = \frac{(\cos x+2)\cos x + \sin^2 x}{(\cos x+2)^2} = \frac{2\cos x+1}{(\cos x+2)^2}$

$\therefore\ \int \frac{2\cos x+1}{(\cos x+2)^2}dx = \frac{\sin x}{\cos x+2} + C\ \therefore\ g(x) = \frac{\sin x}{\cos x+2} + C \Rightarrow g(0) = 0 \Rightarrow C = 0$

$\therefore\ g(x) = \frac{\sin x}{\cos x+2} \Rightarrow g\left(\frac{\pi}{2}\right) = \frac{1}{2}$(rational).

C. Let $x+5 = 14\cos\theta \Rightarrow y-12 = 14\sin\theta\ \therefore\ x^2+y^2 = (14\cos\theta-5)^2 + (14\sin\theta+12)^2$

$= 196+25+144+28(12\sin\theta - 5\cos\theta) = 365+28(12\sin\theta-5\cos\theta)$

$\therefore\ \left.\sqrt{(x^2+y^2)}\right|_{min} = \sqrt{365-28\times13} = \sqrt{365-364} = 1.$

D. $k(x)=\int\frac{(x^2+1)dx}{(x^3+3x+6)^{1/3}}$ put $x^3+3x+6=t^3 \Rightarrow 3(x^2+1)dx=3t^2dt \Rightarrow d(x)=\int\frac{t^2dt}{t}=\frac{t^2}{2}+C$

$k(x)=\frac{1}{2}(x^3+3x+6)^{2/3}+C \Rightarrow k(-1)=\frac{1}{2}(2)^{2/3}+C \Rightarrow C=0 \Rightarrow k(x)=\frac{1}{2}(x^3+3x+6)^{2/3}$;

$f(-2)=\frac{1}{2}(-8)^{2/3}=\frac{1}{2}\left[(-2)^3\right]^{2/3}=2.$

Que. 26. (A) R; (B) S; (C) P; (D) Q

(A) $2\log x-3\log y=1$

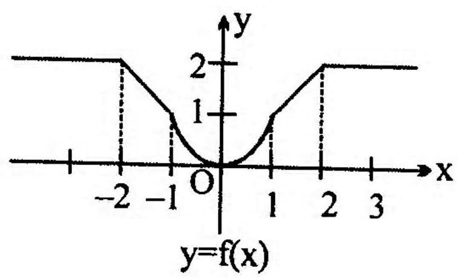
$2\log x+3\log y=7$

—————————

$4\log x=8 \Rightarrow \log x=2; \quad \therefore \log y=1$

$\therefore \quad \log x+\log y=3 \Rightarrow \log(xy)=3$ **Ans.** $\Rightarrow$ (R)

(B) $f(x)=\begin{cases}2 & -5\le x\le -2\\ |x| & 1<|x|<2\\ x^2 & -1\le x\le 1\end{cases}$



(C) $I=\int_0^2\frac{2x^3-6x^2+9x-5}{x^2-2x+5}dx$ (put $x-1=t$ as $x\to 0,\ t=-1$ and $x\to 2,\ t\to 1$)

$=\int_{-1}^{1}\frac{2(1+t)^3-6(1+t)^2+9(1+t)-5}{t^2+4}dx=\int_{-1}^{1}\frac{2t^3+3t}{t^2+4}dx \Rightarrow I=0$ **Ans.** $\Rightarrow$ (P)

(D) Consider $4^{x^2}+4^{(x-1)^2} \Rightarrow AM\ge GM$ for two positive numbers 4^{x^2} and $4^{(x-1)^2}$

$\frac{4^{x^2}+4^{(x-1)^2}}{2}\ge\left[4^{x^2}.4^{(x-1)^2}\right]^{1/2}=2^{x^2}.2^{(x-1)^2}=2^{x^2+(x-1)^2}$

$4^{x^2}+4^{(x-1)^2}\ge 2^{x^2}+(x-1)^2+1$ now $z=x^2+(x-1)^2+1$

$\frac{dz}{dx}=2x+2(x-1)=0$ for maximum or minimum $\Rightarrow x=\frac{1}{2}$

hence $z_{min}=\frac{1}{4}+\frac{1}{4}+1=\frac{3}{2}$

$\therefore \quad 4^{x^2}+4^{(x-1)^2}$ has the minimum value $=2^{3/2}$

hence $f(x)\ge\log_2(2)^{3./2}$

hence $f(x)\ge\log_2(2)^{3/2}=\frac{3}{2}$

$\therefore \quad y\ge\frac{3}{2} \Rightarrow$ range is $\left[\frac{3}{2},\infty\right) \Rightarrow a=\frac{3}{2}$ Ans. $\Rightarrow$ (Q)]

Subjective Type (Up to 4 digit)

Que. 1. $\log_{10}\left(\frac{\sqrt{6-2\sqrt{5}}+\sqrt{6+2\sqrt{5}}}{\sqrt{2}}\right)=\log_{10}\left(\frac{\left(\sqrt{5}-1\right)+\left(\sqrt{5}+1\right)}{\sqrt{2}}\right)=\log_{10}\left(\frac{2\sqrt{5}}{\sqrt{2}}\right)=\log_{10}\sqrt{10}=\frac{1}{2}.$

Alternatively : $x=\sqrt{3-\sqrt{5}}+\sqrt{3+\sqrt{5}}$; $\quad x^2=6+2\sqrt{4}=10 \quad \Rightarrow \quad x=\sqrt{10}$

$\therefore \quad \log_{10}\sqrt{10}=\frac{1}{2}$

Que. 2 $N=\frac{\log_5 250}{\log_{50}5}-\frac{\log_5 10}{\log_{1250}5}$

$N=\left(3+\log_5 2\right)\left(2+\log_5 2\right)-\left(1+\log_5 2\right)\left(4+\log_5 2\right)$

$N=\left(\log_5 2\right)^2+5\log_5 2+6-\left[\left(\log_5 2\right)^2+5\log_5 2+4\right]$

$N=6-4=2.$

Que. 3. Let $K=x^{\frac{1}{\ell n\, y}+\frac{1}{\ell n\, z}}.y^{\frac{1}{\ell n\, z}+\frac{1}{\ell n\, x}}.z^{\frac{1}{\ell n\, x}+\frac{1}{\ell n\, y}}$

$\ell n\, K=\ell n\, x\left[\frac{1}{\ell n\, y}+\frac{1}{\ell n\, z}\right]+\ell n\, y\left[\frac{1}{\ell n\, z}+\frac{1}{\ell n\, x}\right]+\ell n\, z\left[\frac{1}{\ell n\, x}+\frac{1}{\ell n\, y}\right]$ ....(1)

Put $\ell n\, x+\ell n\, y+\ell n\, z=0$ (given)

$\therefore \quad \frac{\ell n\, x}{\ell n\, y}+\frac{\ell n\, z}{\ell n\, y}=-1; \quad \frac{\ell n\, y}{\ell n\, x}+\frac{\ell n\, z}{\ell n\, x}=-1$ and $\frac{\ell n\, x}{\ell n\, z}+\frac{\ell n\, y}{\ell n\, z}=-1$

$\therefore$ RHS of $(1)=-3 \quad \Rightarrow \quad \ell nk=-3 \quad \Rightarrow \quad k=e^{-3}.$

Que. 4. $L=\text{anti}\log_{32}0.6=(32)^{6/10}=2^{\frac{5\times 6}{10}}=2^3=8$

$M=\text{Integer from } 625 \text{ to } 3125=2500$

$N=49^{(1-\log_7 2)}+5^{-\log_5 4}=49\times 7^{-2\log_7 2}+5^{-\log_5 4}=49\times\frac{1}{4}+\frac{1}{4}=\frac{50}{4}=\frac{25}{2}$

$\therefore \quad \frac{LM}{N}=\frac{8\times 2500\times 2}{25}=1600.$

Que. 5. $(-5,-4)\cup(-3,-1)$ $\;2\log_{1/4}(x+5)>\frac{9}{4}\log\frac{1}{3\sqrt{3}}(9)+\log\sqrt{x+5}(2)$

$-\log_2(x+5)>\frac{9}{4}\left(-\frac{4}{3}\right)+\frac{2}{\log_2(x+5)}\left(\log_{3^{-3/2}}(9)=t\Rightarrow 3^{-\frac{3t}{2}}\Rightarrow-\frac{3t}{2}=2\Rightarrow t=-\frac{4}{3}\right)$

$3>\log_2(x+5)+\frac{2}{\log_2(x+5)}$

Let $\quad \log_2(x+5)=y$

$3>y+\frac{2}{y} \quad \Rightarrow \quad 3>\frac{y^2+2}{y} \quad \Rightarrow \quad \frac{y^2+2}{y}-3<0 \quad \Rightarrow \quad \frac{y^2-3y+2}{y}<0$

$$\frac{(y-2)(y-1)}{y}<0$$

$y\in(-\infty,0)\cup(1,2)$

$\therefore \quad -\infty<\log_2(x+5)<0$

$0<x+5<1 \quad \Rightarrow \quad -5<x<-4$

Or $\quad 1<\log_2(x+5)< \quad \Rightarrow \quad 2<x+5<4$

$\therefore \quad$ S is $(-5,-4)\cup(-3,-1)$

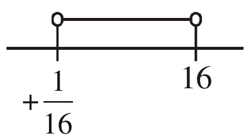
Que. 6. (3072)

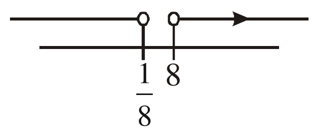
$\left(\log_2^2 x\right)^2-(5\log_2 x-2)^2-20\log_2 x+148<0 \Rightarrow \left(\log_2^2 x\right)^2-\left(25\log_2^2 x+4-20\log_{10} x\right)-20\log_2 x+148<0$

put $\quad \log_2^2 x=y \Rightarrow y^2-25y-4+148<0 \Rightarrow y^2 25y+144<0 \Rightarrow (y-16)(y-5)<0 \Rightarrow 9<y<16$

$\therefore\ 9<(\log_2 x)^2<16$ If $\log_2^2 x-16<0 \Rightarrow (\log_2 x-4)(\log_2 x+4)<0$ If $(\log_2 x)^2-9>0$

$\Rightarrow(\log_2 x-3)-(\log_2 x+3)>0$



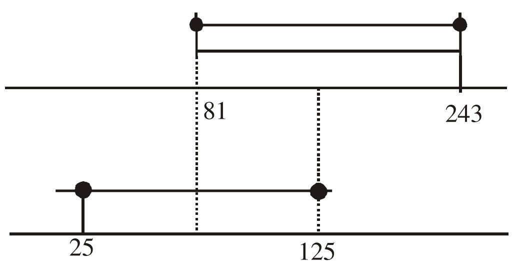


Hence $x\in\left(\frac{1}{16},\frac{1}{8}\right)\cup(8,16)$. Hence $x\in(8,16)$; $\therefore\ a=8; b=16 \Rightarrow ab(a+b)=3072.$

Que. 7. Consider $5^{-p}=5^{-\log_5(\log_5 3)}=\frac{1}{5^{\log_5(\log_5 3)}}=\frac{1}{\log_5 3}\ \therefore 5^{-p}=\frac{1}{\log_5 3}=\log_3 5$ now $3^{C+\log_3 5}=405$

$\Rightarrow 3^C.3^{\log_3 5}=405; 3^C.5=405 \Rightarrow 3^C=81=3^4 \Rightarrow C=4$

Que. 8. (i) $\left.\begin{matrix}\alpha_1=4\Rightarrow 3^4\le N<3^5\\ \alpha_2=2\Rightarrow 5^2\le N<5^3\end{matrix}\right\}\Rightarrow 81\le N<125$ No. of integral values of N = 125 – 81 = 44.



(ii) $\alpha_1=5\Rightarrow 3^5\le N<3^6\Rightarrow 243\le N<729$

$\alpha_2=3\Rightarrow 5^3\le N<5^4\Rightarrow 125\le N<625$

$\alpha_3=2\Rightarrow 7^2\le N<7^3\Rightarrow 49\le N<343$

$\therefore\ 343\le N<343\Rightarrow N_{max}=342$

(iii) Difference 342 – 243 = 99.

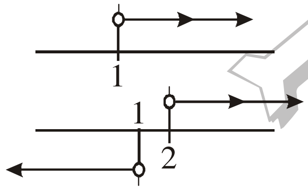
Que. 9. (0203.00) $\quad \log_{10}(2xy)=2\Rightarrow 2xy=100$ ....(1) also $\log_{10}\left(\frac{x^2}{2y}\right)=4\Rightarrow\frac{x^2}{2y}=10^4$ ....(2)

From (1) and (2) $\frac{x^2.x}{100} = 10^4 \Rightarrow x^3 = 10^6 \Rightarrow x = 100$ and $y = \frac{1}{2}$ $\therefore$ $x + y = \frac{201}{2} \Rightarrow m + n = 203.$

Que. 10. nequality is ture if $0 \le \log_2\left(\frac{2x-3}{x-1}\right) < 1$ i.e. $1 \le \frac{2x-3}{x-1} < 2$, let, $\frac{2x-3}{x-1} - 2 < 0 \Rightarrow \frac{2x-3-2x+2}{x-1} < 0$

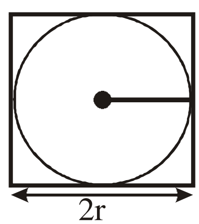
$\Rightarrow \frac{-1}{x-1} < 0 \quad \Rightarrow \quad \frac{1}{x-1} > 0 \quad \Rightarrow \quad x > 1 \qquad$ ....(1)

and $\frac{2x-3}{x-1} \ge 1 \Rightarrow \frac{2x-3}{x-1} - 1 \ge 0 \Rightarrow \frac{2x-3-x+1}{x-1} \ge 0 \Rightarrow \frac{x-2}{x-1} \ge 0 \Rightarrow x \ge 2$ or $x < 1$ ....(2)

$\Rightarrow$ 

taking intersection of (1) and (2) $x \ge 2$ i.e. $[2, \infty)$

Que. 11. (864) $\quad V = \pi r^2 h, \quad V = 1728, \quad S = 2\pi rh = 8r^2 \quad \Rightarrow \quad S(r) = 8r^2 + 2\pi r.\frac{V}{\pi r^2} = 8r^2 + \frac{2V}{r}$



$S'(r) = 16r - \frac{2V}{r^2} = 0 \quad \Rightarrow \quad r^3 = \frac{2V}{16} = \frac{V}{8} = \frac{1728}{8} = 216 \;\Rightarrow\; S(r)\big|_{min} = 8.36 + \frac{2.1728}{6} = 288 + 576 = 864.$

Que. 12. (8550). $\quad {}^{10}C_2.{}^{20}C_2 = 45 \times 190 = 8550.$

Que. 13. (66) $\quad$ Sub. $\begin{array}{r} 7xy + 6y + 2z = 272 \\ 2x - 2y + 2z = 32 \\ \hline 5x + 8y = 240 \end{array} \Rightarrow \quad x = \frac{240-8y}{5} = 48 - \frac{8}{5}y$ let $y = 5k, k \in I$ $\therefore$ $x = 48 - 8k$

$\therefore \; x - y + z = 16 \qquad \Rightarrow \qquad (48-8k) - 5k + z = 16 \;\Rightarrow\; z = 13k - 32 > 0 \;\Rightarrow\; k > \frac{32}{13} \;\Rightarrow\; k \ge 3$

Now $\; 48 - 8k > 0 \Rightarrow k < 6 \Rightarrow k \le 5 \;\therefore\; 3 \le k \le 5 \Rightarrow k = 5 \;\therefore\; Z_{max} = 65 - 32 = 33$

$\Rightarrow y = 25; x = 8 \qquad \therefore \; x, y, z \equiv (8, 25, 33) \;\Rightarrow\; \text{sum} = 66.$

Que. 14. (343) $\quad \vec{V} = \vec{A} \times \left(\left(\vec{A}.\vec{B}\right)\vec{A} - \left(\vec{A}.\vec{A}\right)\vec{B}\right).\vec{C} = \left(\underbrace{\vec{A} \times \left(\vec{A}.\vec{B}\right)\vec{A}}_{\text{zero}} - \left(\vec{A}.\vec{A}\right)\vec{A} \times \vec{B}\right).\vec{C} = -\left|\vec{A}\right|^2 \left[\vec{A}\;\vec{B}\;\vec{C}\right]$

Now $\left|\vec{A}\right|^2 = 4 + 9 + 36 = 49 \;\Rightarrow\; \left[\vec{A}\;\vec{B}\;\vec{C}\right] = \begin{vmatrix} 2 & 3 & 6 \\ 1 & 1 & -2 \\ 1 & 2 & 1 \end{vmatrix} = 2(1+4) - 1(3-12) + 1(-6-6)$

$= 10 + 9 - 12 = 7 \qquad \therefore \qquad \left|-\left|\vec{A}\right|^2 \left[\vec{A}\;\vec{B}\;\vec{C}\right]\right| = 49 \times 7 = 343.$